Time-Frequency (Wavelet) Methods for the Analysis of Shock Wave Boundary Layer Interactions

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Data from Skytop Lab at SU, from Spectral Energies LLC, and from DLR Stuttgart

Thanks to M. Glauser, AS Magstadt, MG Berry, TJ Coleman, CJ Ruscher, I. Boxx

and many others

Support from AFOSR and AFRL/Spectral Energies

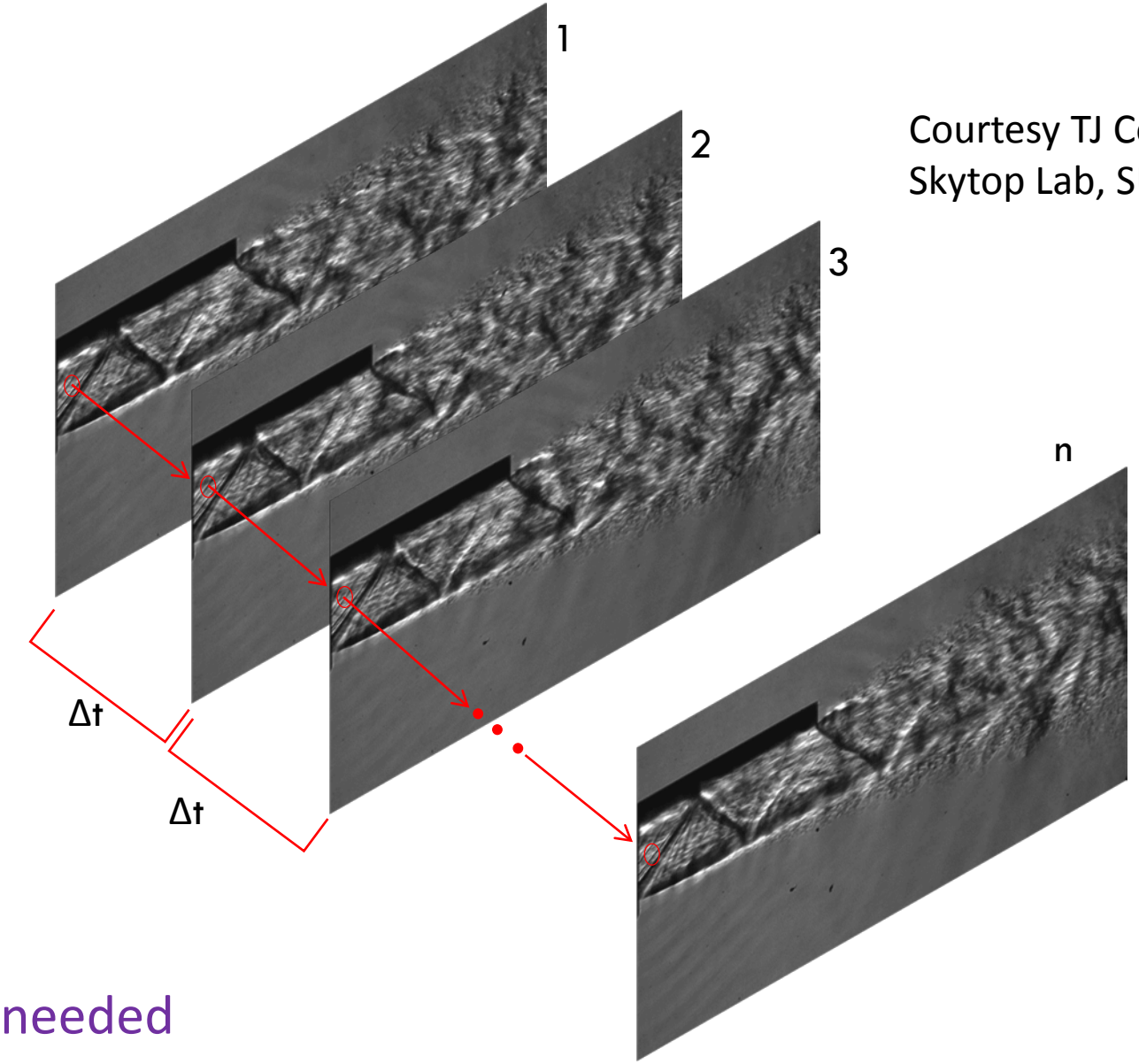
Methods (Lewalle) and results (Kan)

- Survey of continuous wavelet (time-frequency) methods
  - Band-pass filtering
  - Energy spectra
  - Cross-correlations, frequency dependence
  - Phase averaging
  - Pattern recognition: contributing events
- Selected examples from
  - Supersonic jets: experimental (100 kHz Schlieren, SU Skytop Lab)
  - Supersonic jets: LES (Chris Ruscher, Spectral Energies LLC)
  - Swirl-stabilized burner (Isaac Boxx, DLR Stuttgart)
  - No SWBLI here, but the *methods* seem relevant (related talks in this workshop)

  - Multiscale, intermittent: think ‘wavelets’

Time series

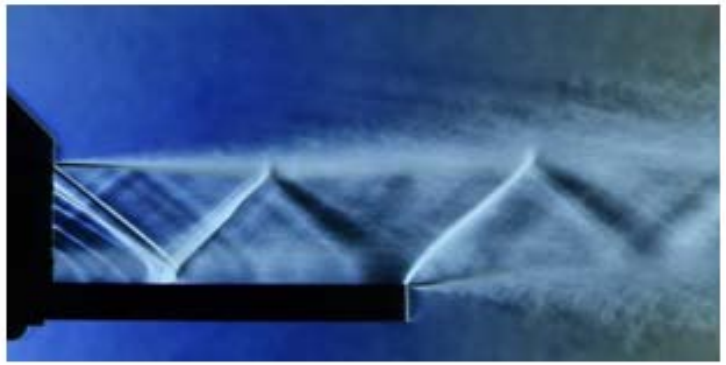
- Acoustic
- Wall sensors
- Computational
- PIV
- Schlieren
- Multi-point, multi-sensor, simultaneous
- Similar methods applicable in space, but time domain is assumed



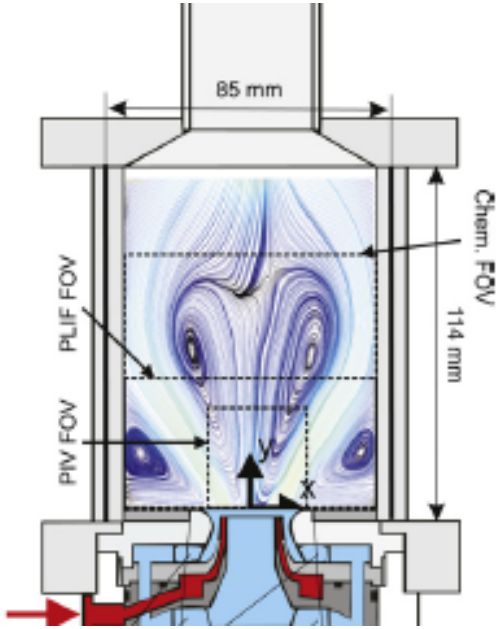
Courtesy TJ Coleman, Skytop Lab, SU

Extensive post-processing needed to generate time series

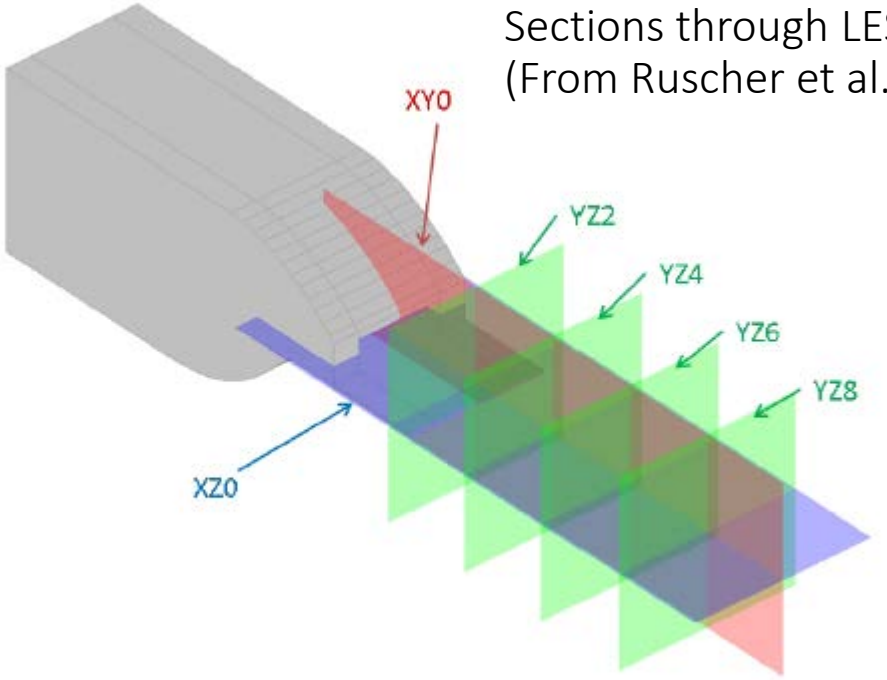
Convert spatial fields to time series



High-speed Schlieren images (courtesy Magstadt and Berry, Skytop Lab)

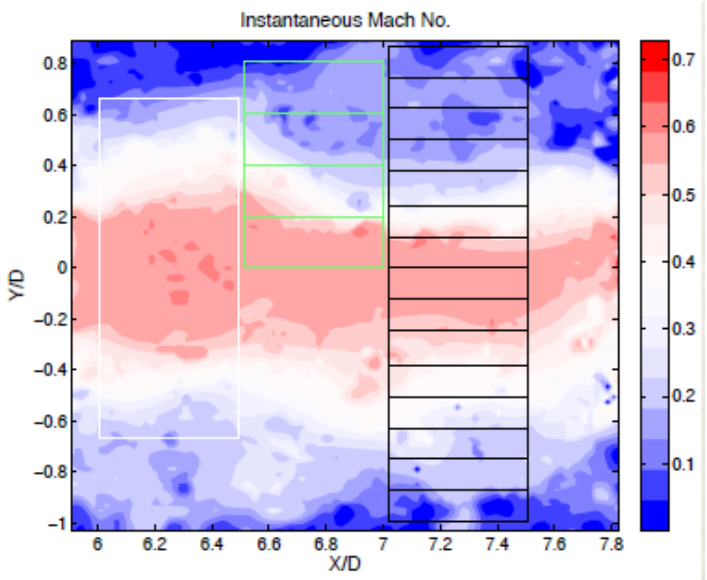


3-component PIV, swirl-stabilized burner (from Steinberg et al. 2010)



Sections through LES pressure data (From Ruscher et al., JPC 2015)

Near-jet PIV data (From Kan et al, AIAA SciTech 2015)



Fourier transforms

Complex wave:

$$w = e^{2i\pi\omega t}$$

Transform: projection of f(t) on the complex wave

$$\hat{f}(\omega) = (f, w^*) = \int f(t) e^{-2i\pi\omega t} dt$$

Real signals:

$$\hat{f}(-\omega) = \hat{f}^*(\omega)$$

Inverse transform

$$f(t) = \int \hat{f}(\omega) e^{2i\pi\omega t} d\omega$$

Parseval/Plancherel – f(t) and g(t) real

$$\int f(t) g(t) dt = \int \hat{f}(\omega) \hat{g}^*(\omega) d\omega$$

Particular case

$$\int \mid f(t) \mid^2 dt = \int \mid \hat{f}(\omega) \mid^2 d\omega$$

The signal's energy is distributed among frequencies

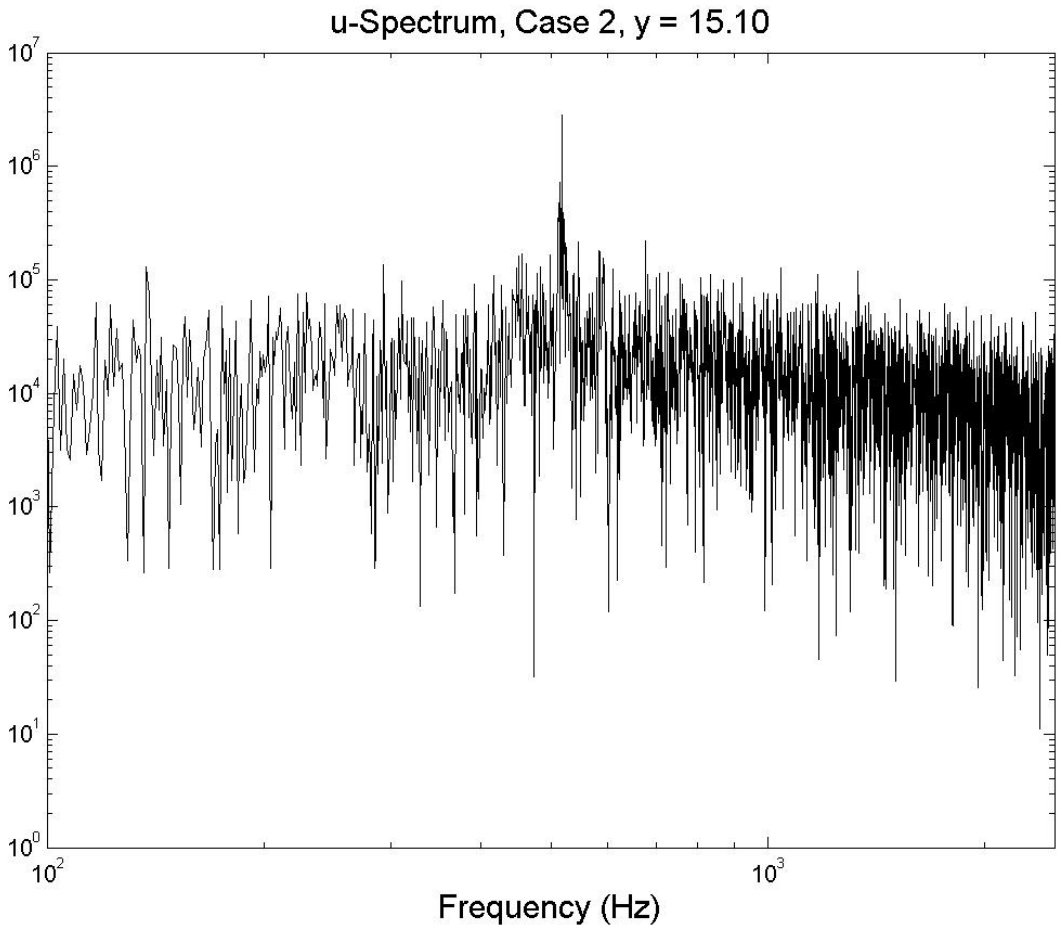
Fourier spectra

Spectrum: start from Plancherel

$$E = \int e(t)dt = \int e_F(\omega)d\omega$$
$$e(t) = \frac{1}{2} \mid f(t) \mid^2$$
$$e_F(\omega) = \frac{1}{2} \mid \hat{f}(\omega) \mid^2$$

Compensated spectrum: same information in logarithmic scale

$$E = \int e(t)dt = \int e_c(\omega)\frac{d\omega}{\omega}$$
$$e_c(\omega) = \omega e_F(\omega)$$



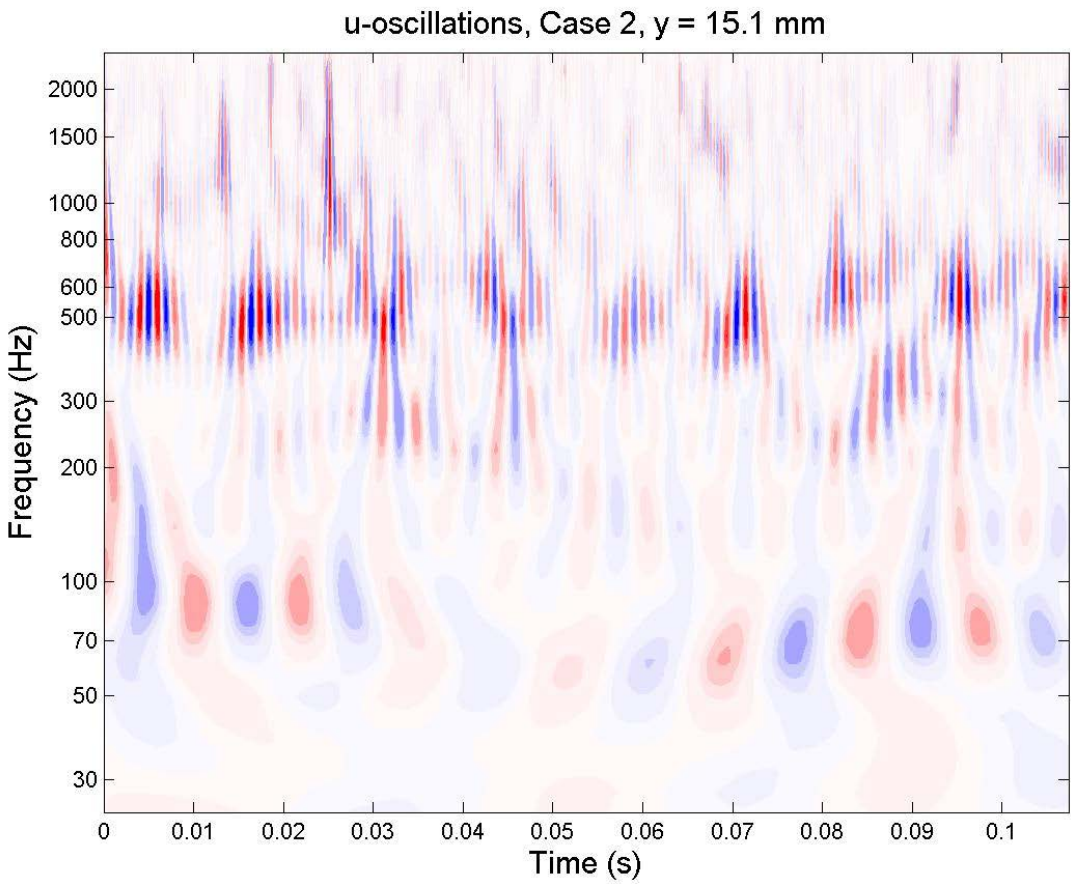
Drawback: short records → noisy spectrum
Advantage: periodicity shows!

Continuous wavelet transforms

Continuous wavelet $\psi(t)$

- admisssible
- convolution with the signal
- change of scale: dilation factor a

$$\tilde{u}(a,t) = \int_{-\infty}^{\infty} u(t') \, \frac{1}{\sqrt{a}} \psi^*\left(\frac{t-t'}{a}\right) dt'$$



A long history in fluid mechanics

Any wavelet transform is a **band-pass filter**

$$\tilde{u}(a,t) = \int_{-\infty}^{\infty} u(t') \, \frac{1}{\sqrt{a}} \psi^*\left(\frac{t-t'}{a}\right) dt'$$

$$= \int \hat{u}(\omega) \, \frac{1}{\sqrt{a}} \hat{\psi}(\omega a) \, e^{2i\pi\omega t} d\omega$$

Inverse Fourier

Weight

Choice of a wavelet = choice of its passing band

The triad **Transform / Inverse / Parseval** makes all the difference

Inverse transform and Parseval

Inverse transform

$$u(t) = \frac{1}{c_\psi} \int_{-\infty}^{\infty} \int_0^{\infty} [\tilde{u}(a,t') \frac{1}{\sqrt{a}} \psi(\frac{t-t'}{a})] \frac{da}{a^2} dt'$$

Therefore: no loss of information

Interpretation: the signal is (also) made of elementary blocks ψ at many different scales.

Furthermore, **Parseval**: correlations and energy

$$\int_{-\infty}^{\infty} [u(t) v^*(t)] dt$$

$$= \frac{1}{c_\psi} \int_{-\infty}^{\infty} \int_0^{\infty} \tilde{u}(a,t) \tilde{v}^*(a,t) \frac{da}{a^2} dt,$$

and in particular

$$\int_{-\infty}^{\infty} \mid u(t) \mid^2 dt = \frac{1}{c_\psi} \int_{-\infty}^{\infty} \int_0^{\infty} \mid \tilde{u}(a,t) \mid^2 \frac{da}{a^2} dt,$$

My version (see Springer Handbook, 2007)

Let us simplify!

- Factors $\sqrt{a}$, $1/a^2$, etc.
- Factors c_ψ
- Logarithmic frequencies $(\int \frac{\mathrm{d}\omega}{\omega})$

My version

- Definition of an equivalent frequency instead of the dilation factor
- Absorb these factors through a change in variables in the triad of formulas
- Make sure that the triad is consistent!!

Morlet, the original wavelet: local periodicity

The Morlet wavelet

Complex wave (Fourier: frequency resolution) localized in time by a Gaussian envelope:

$$\psi_{M,z_0}(t) = e^{-2\pi^2 t^2/z_0^2}\,(e^{2i\pi t} - e^{-\frac{z_0^2}{2}})$$

Correction (admissibility) is often omitted $z_0 > 5$

With the factors as above, I take

$$\Psi_M(t,\omega) = \frac{\omega}{\sqrt{c_\psi}}\psi_{M,z_0}(\omega t).$$

ω is the frequency of the wave packet (!!)

Triad of formulas for Morlet

Transform

$$\tilde{u}_M = \int u(t')\,\Psi_M^*((t-t')\omega)dt'$$

Inverse

$$u(t) = \int\int \Psi_M((t-t')\omega)\,\tilde{u}_M(t',\omega)\frac{d\omega}{\omega}dt$$

Parseval

$$\int u\,v\,dt = \int\int \tilde{u}_M\,\tilde{v}_M^*\,\frac{d\omega}{\omega}dt$$

In particular

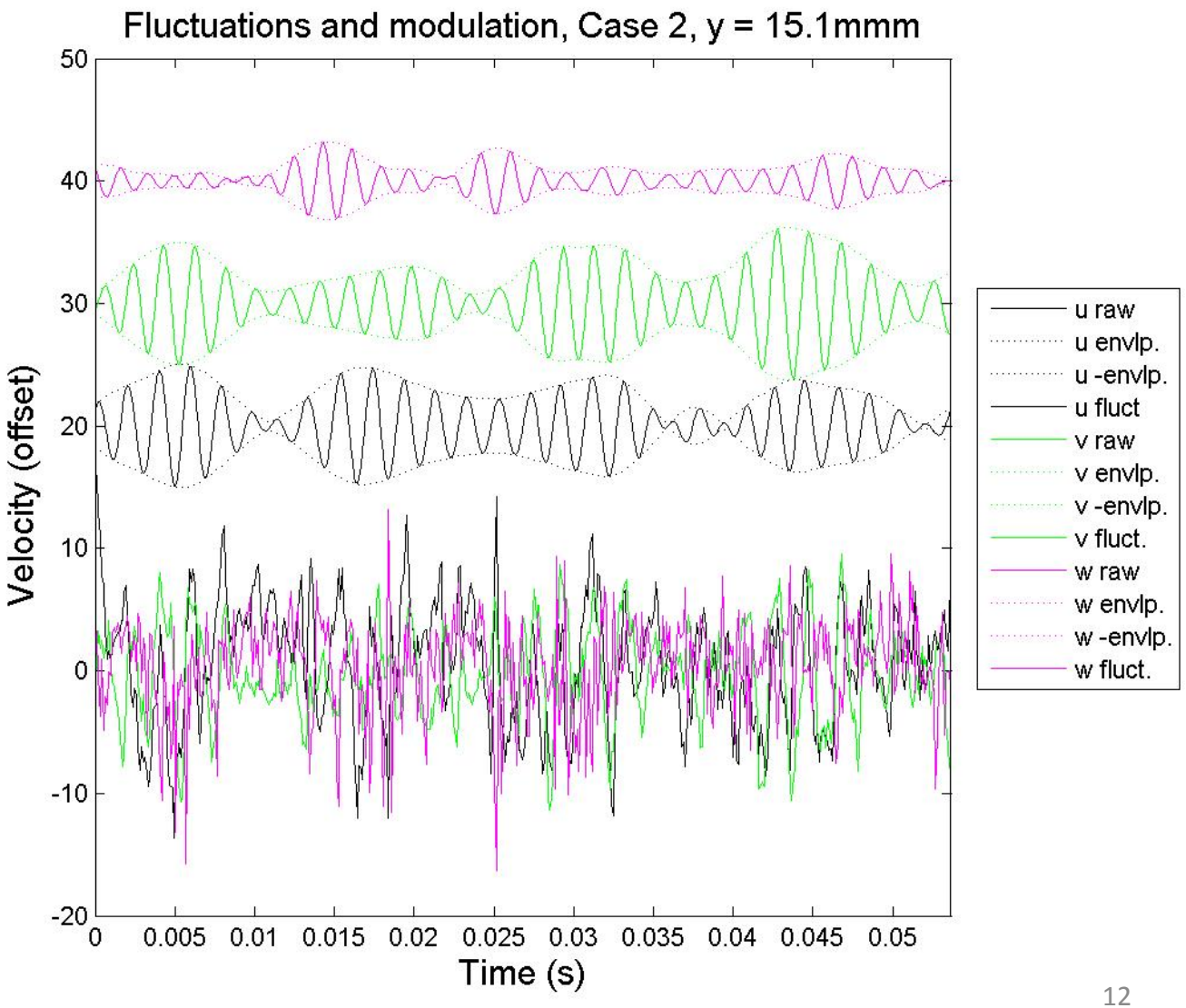
$$\int \frac{u^2}{2}dt = \frac{1}{2}\int\int |\,\tilde{u}_M\,|^2\,\frac{d\omega}{\omega}dt$$

Modulation

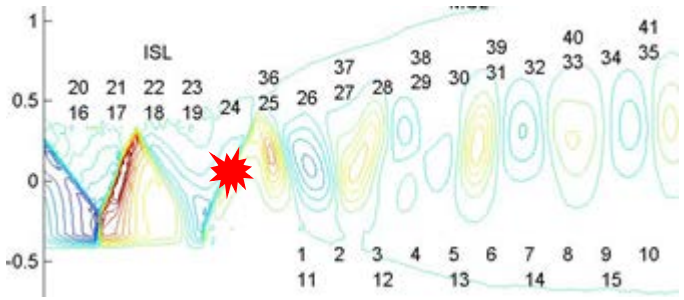
The Morlet transform

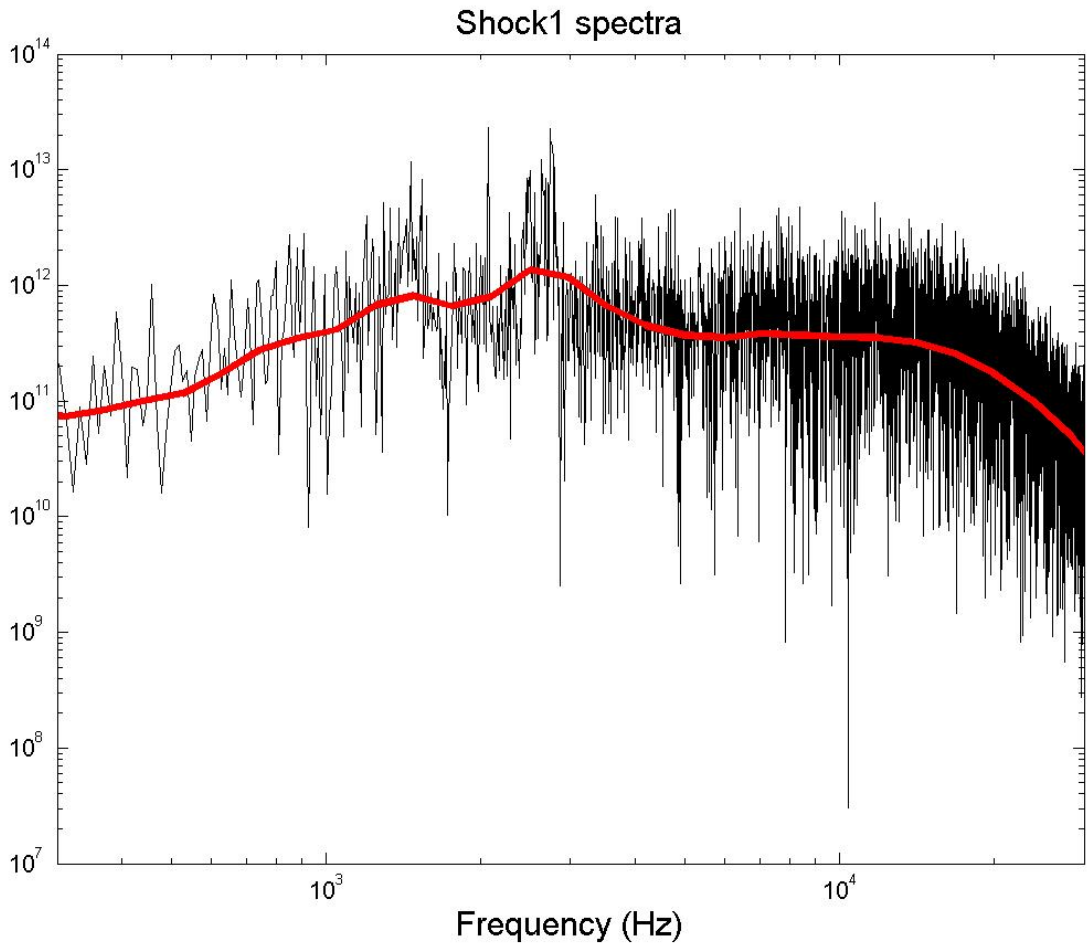
$$\psi_{M,z_0}(t) = e^{-2\pi^2 t^2/z_0^2}\ e^{2i\pi t}$$

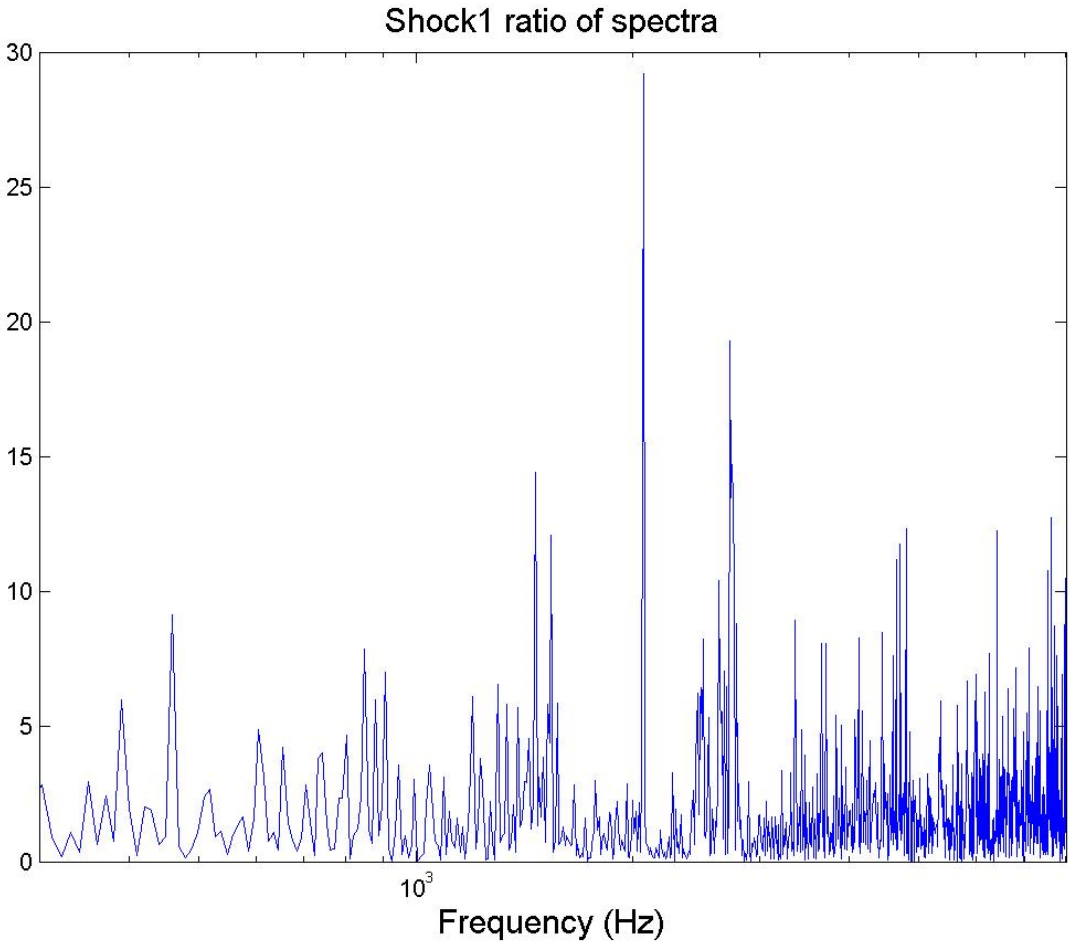
$$\text{Norm} \rightarrow e^{-2\pi^2 t^2/z_0^2} \quad \text{(envelope)}$$
$$\text{Real part} \rightarrow \cos 2\pi t \quad \text{(oscillation)}$$
$$\text{Phase} \rightarrow 2\pi t \ \ mod(2\pi)$$



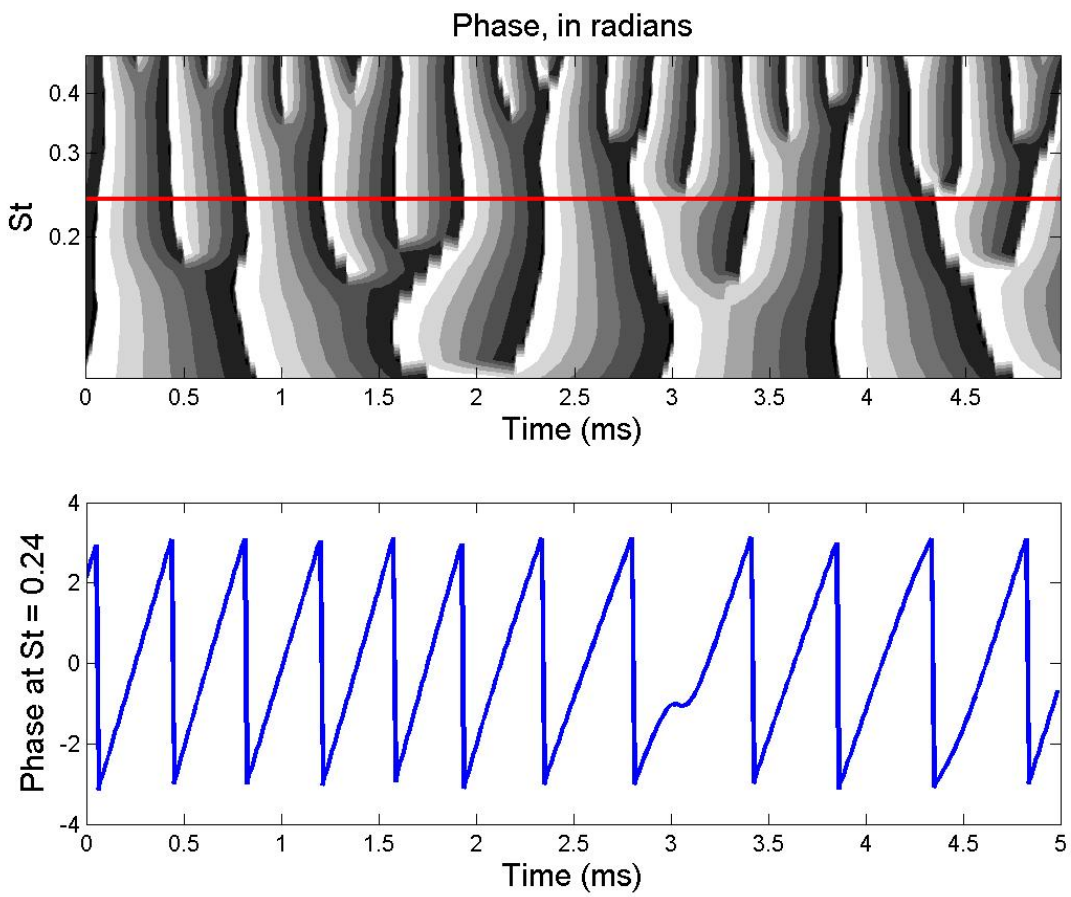
Fourier and wavelet spectra



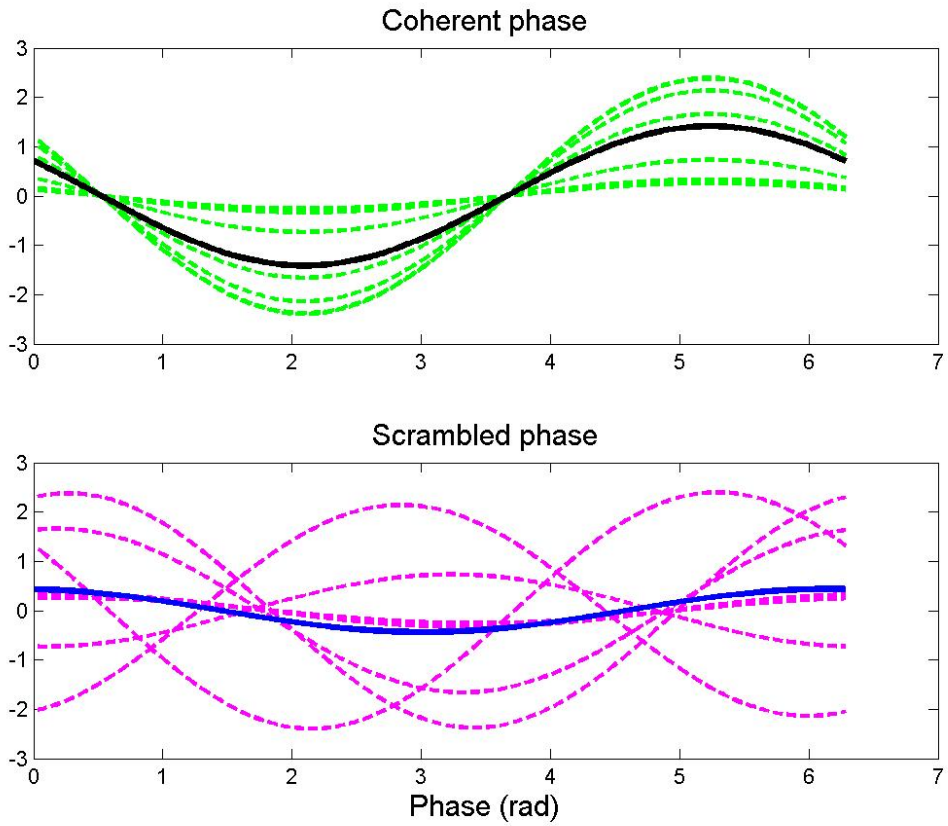




Phase



Phase averaging of normalized signals



Cross-correlation and propagation

Cross-correlation

Signals are normalized (0,1)

Fourier version

$$x(\tau) = \frac{1}{T}\int u(t)v(t+\tau)dt = \frac{1}{T}\int \hat{u}^*(\omega)\hat{v}(\omega)e^{2i\pi\omega\tau}d\omega$$

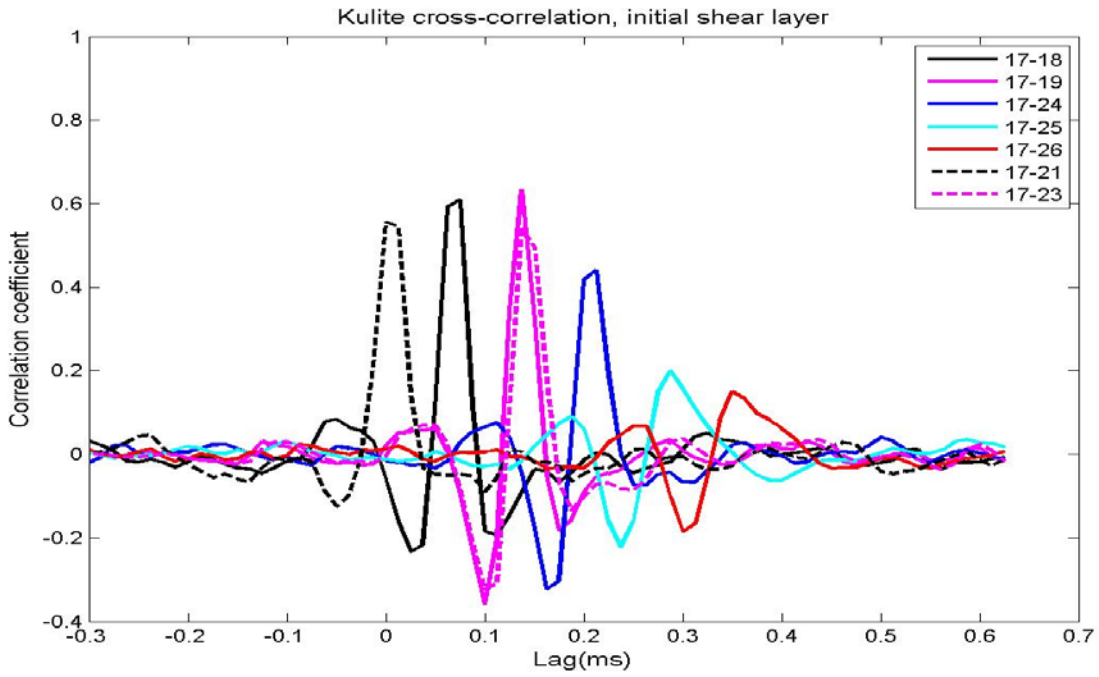
Morlet version

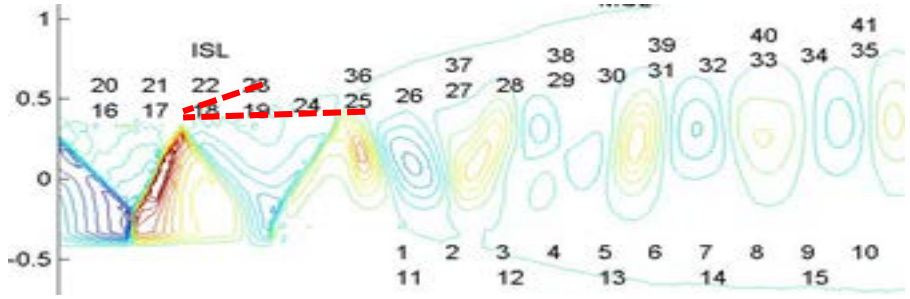
$$\int u(t)v(t+\tau)dt = \frac{1}{T}\int\int \tilde{u}_M^*\ \tilde{v}_M\ \frac{d\omega}{\omega}dt$$

$$= \int X(\omega,\tau)\frac{d\omega}{\omega}$$

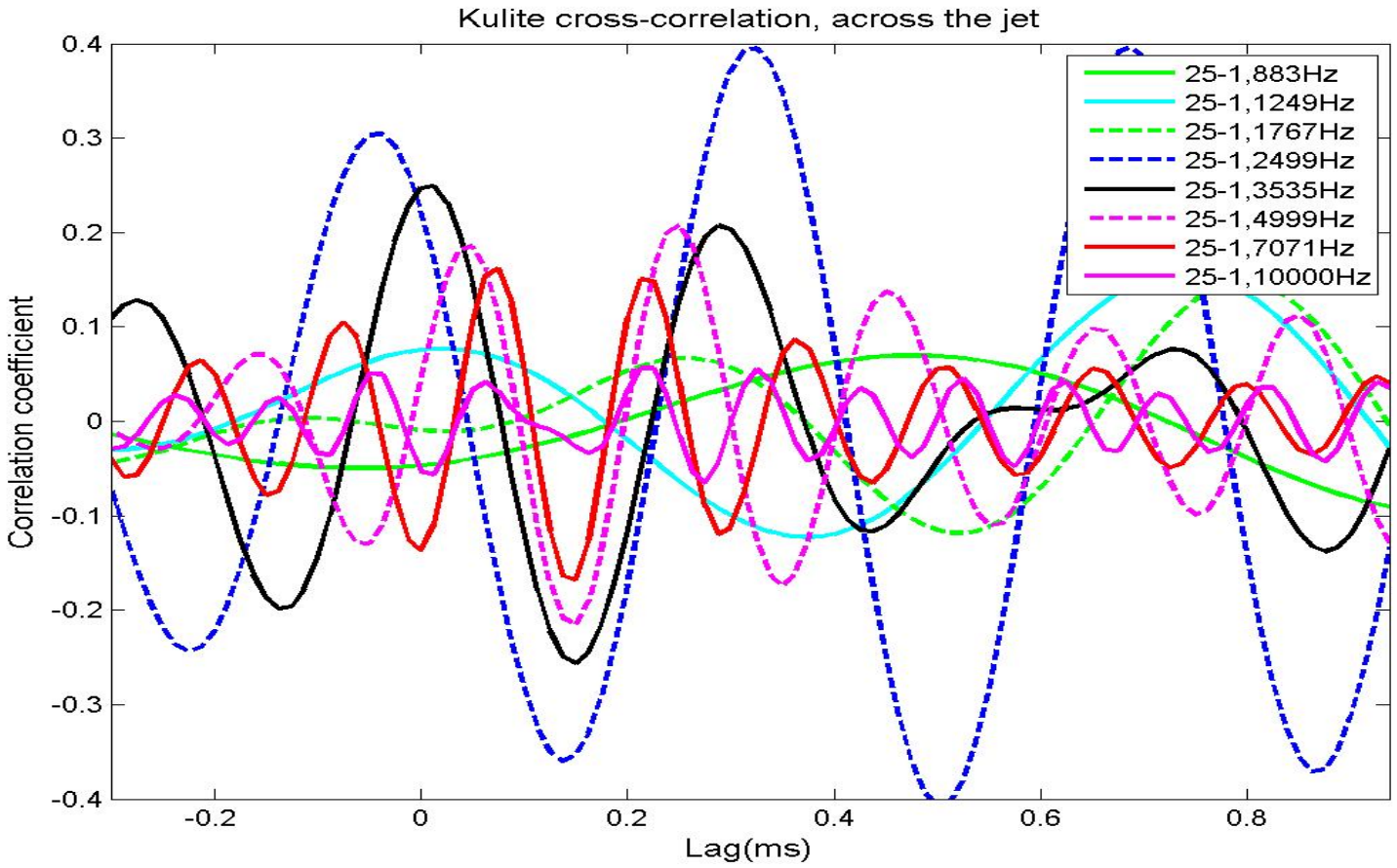
where

$$X(\omega,\tau) = \frac{1}{T}\int \tilde{u}_M^*(\omega,t)\ \tilde{v}_M(\omega,t+\tau)\ dt$$

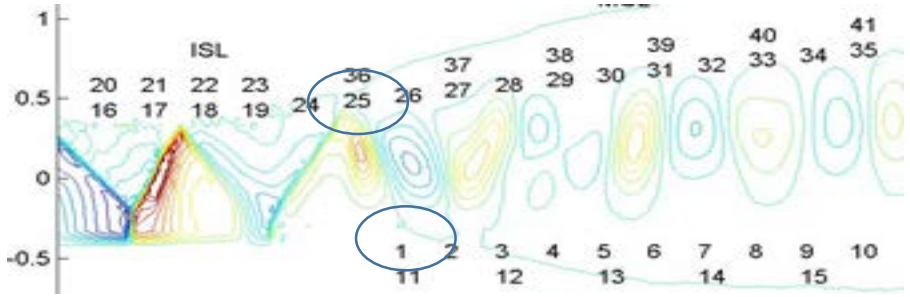




Across the jet



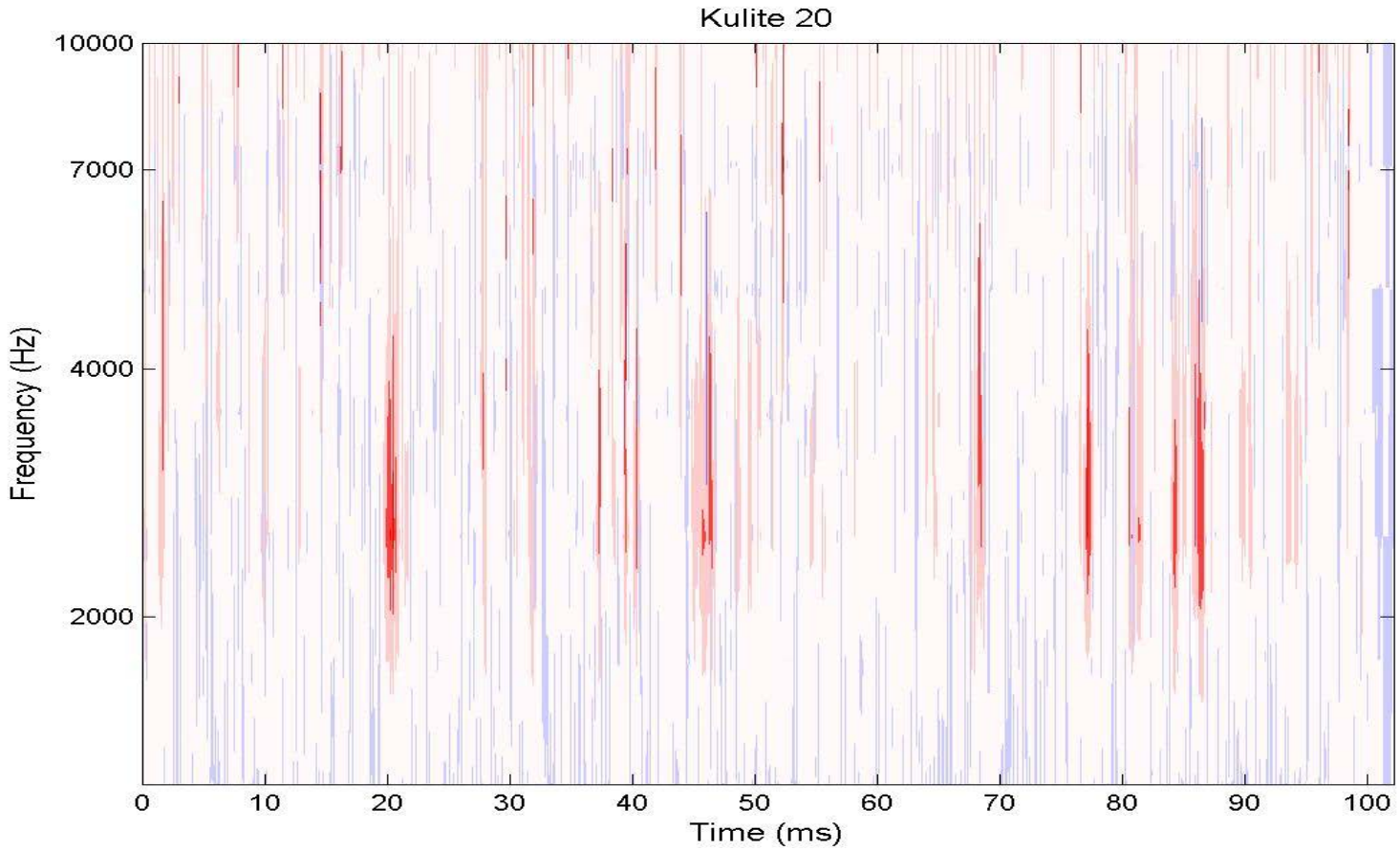
- 2500 Hz dominates!
- Why such a lag??



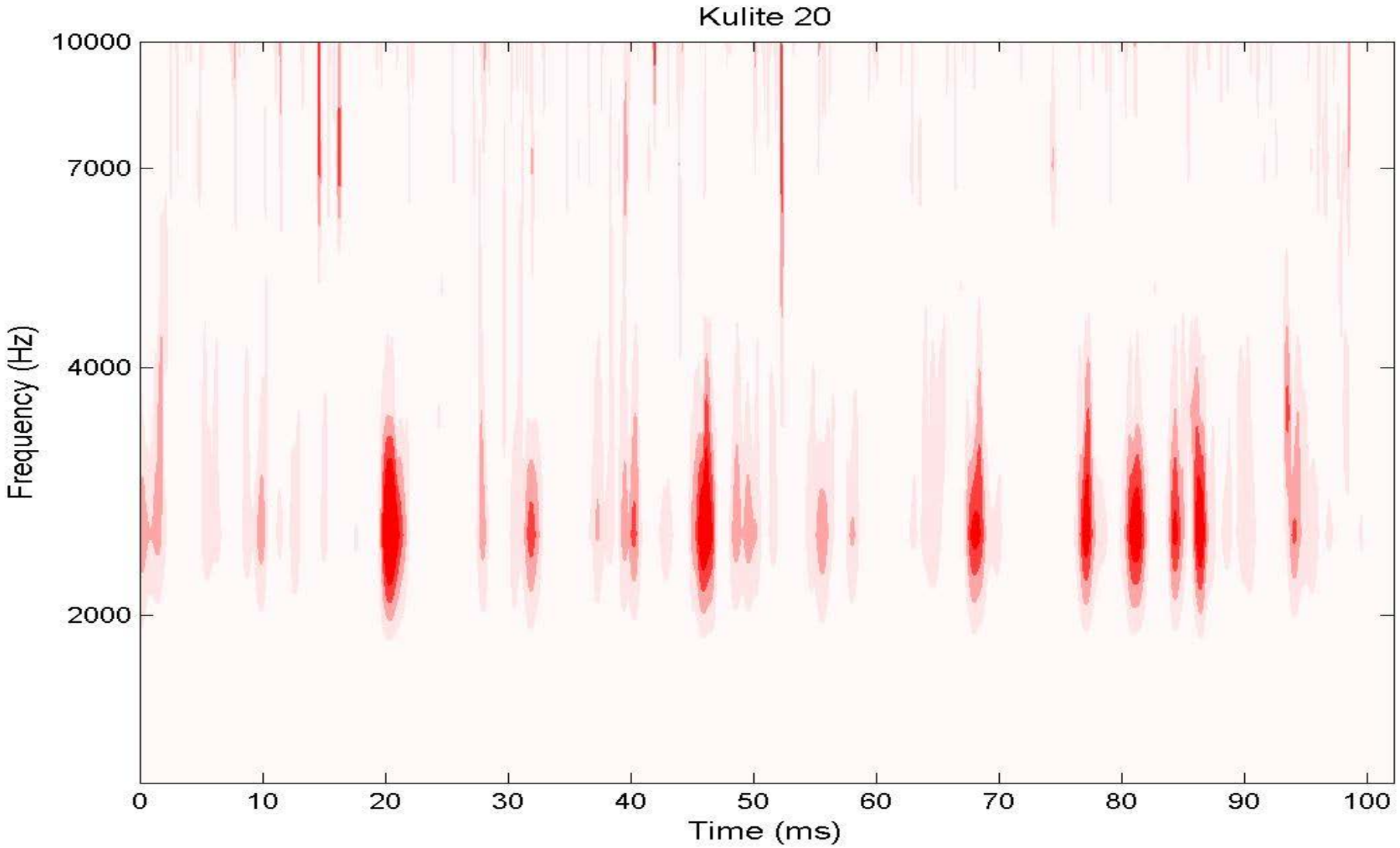
Event level analysis

- Scalograms: positive and negative values
- Product of scalograms, with lags
- Patches in time-frequency: if + and – match, product is positive, otherwise not
- Try and get everything to be as positive as can be
- Lags differ by event
- Time-Frequency-Lag method (TFL)

Best mean lag



Best individual lags



These are some of our tools

Illustrations will be presented by Pingqing Kan

Thank you