LES and Tensor Analysis on Shock-Vortex Ring Interaction

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Contents

What Does Mean
Shock-Turbulent Boundary Layer Interaction?

SBLI means shock-vortex interaction since:

1. Turbulence is a buildup of vortices (most boundary layers are tu rbulent)

2. Vortex rotating very fast and shock can not break through

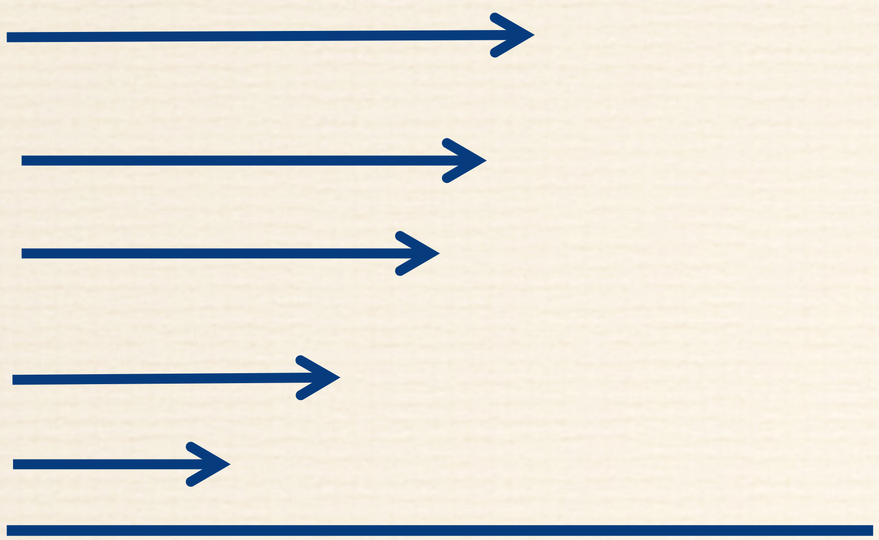
3. Vortices keep moving to cost shock unsteady

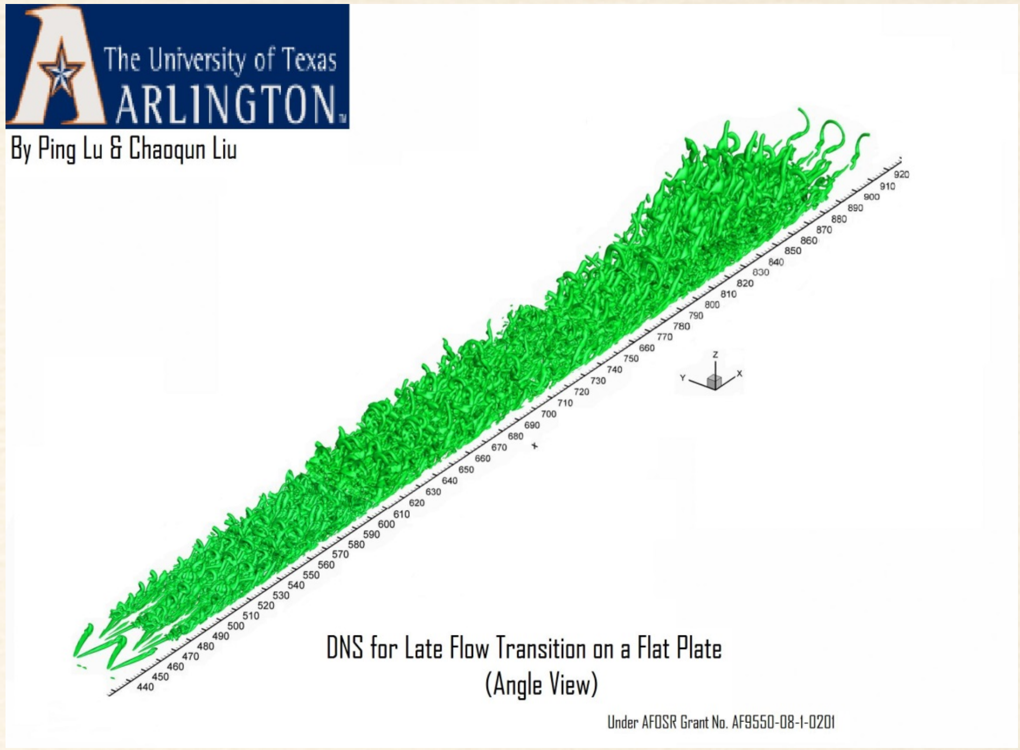
4. Shock moving costs fluctuation of pressure, etc.

5. Fluctuation frequencies are determined by vortices moving speed and vortex shape and size

Please look the sonic lines and shock structure inside boundary layer

What is difference between laminar and turbulent boundary layers?





Blasius – large vorticity but no vortex

Turbulent – vorticity and vortices

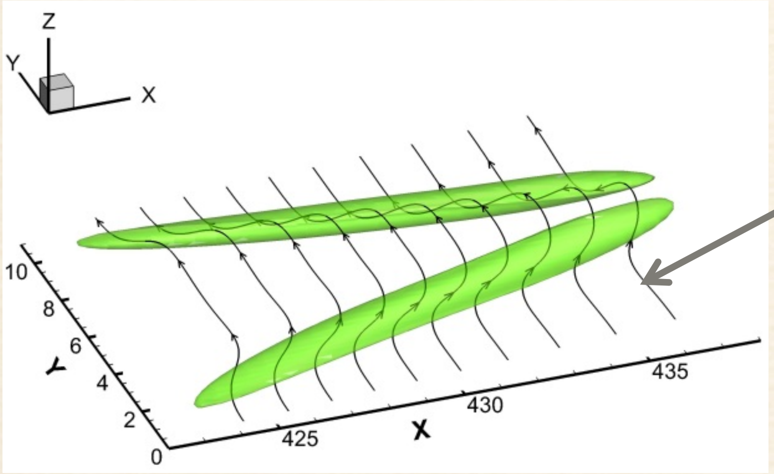
1.Vorticity, vorticity line and vorticity tube have rigorous mathematical definition and there is no much room for us to discuss

1) **Vorticity :** Curl of velocity $\nabla \times \vec{V}$

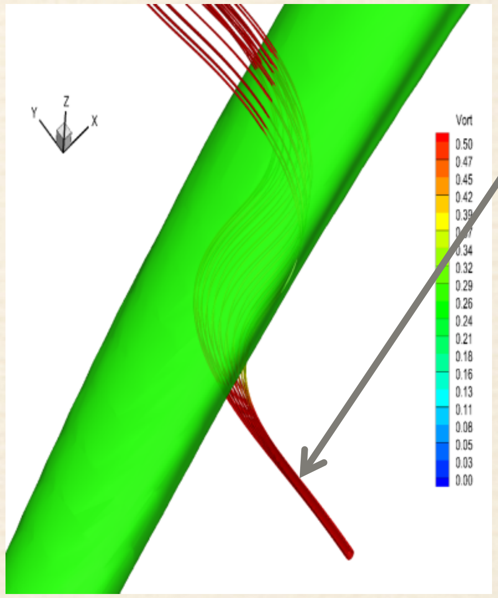
2) **Vorticity Line:** The tangent direction of the line is the vorticity vector direction at each point

3) **Vorticity Tube:** A tube bounded by vorticity lines without vorticity line leak

Vorticity tube



Vorticity lines



How about Vortex??

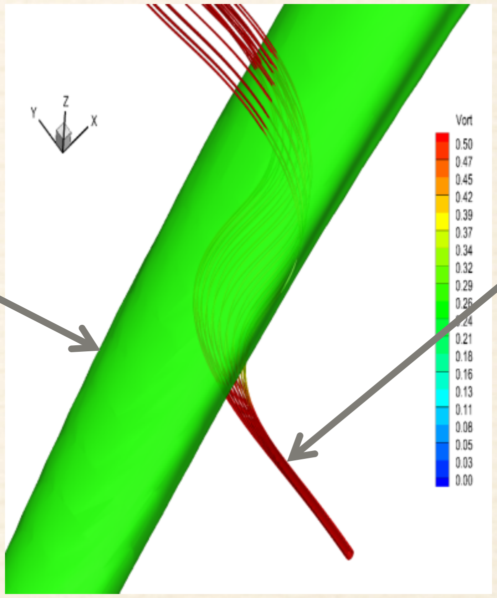
2. Vortex Definition

1. Vorticity tube (web lecture notes, 2006): No
2. Vorticity line congregation (Vortex dynamics, 2007): No

3. Vortex is a region where vorticity overtakes deformation

- My definition

Vortex

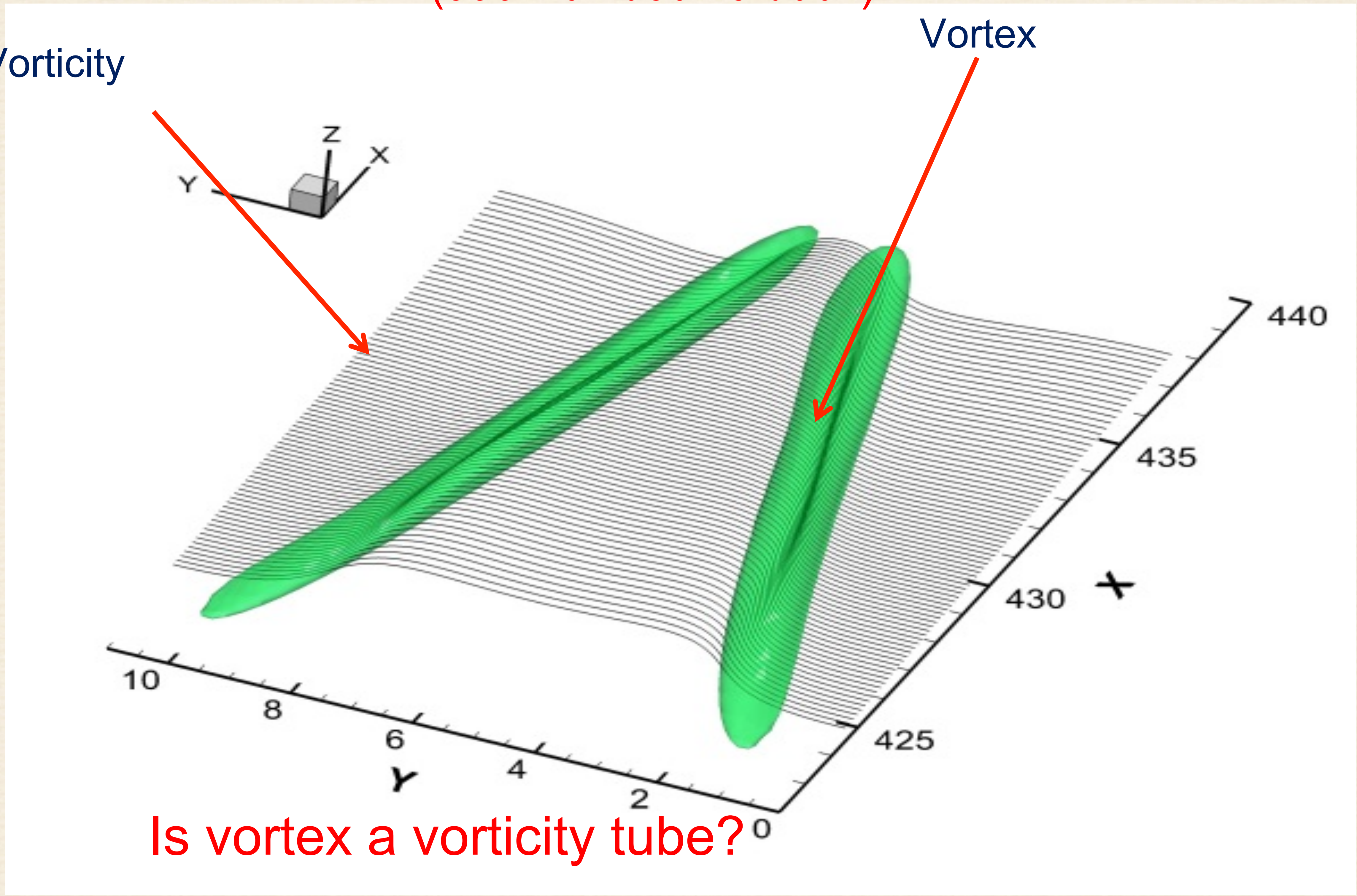


Vorticity Tube

Vortex is vorticity tube? No

Vortex is congregation of vorticity lines? No

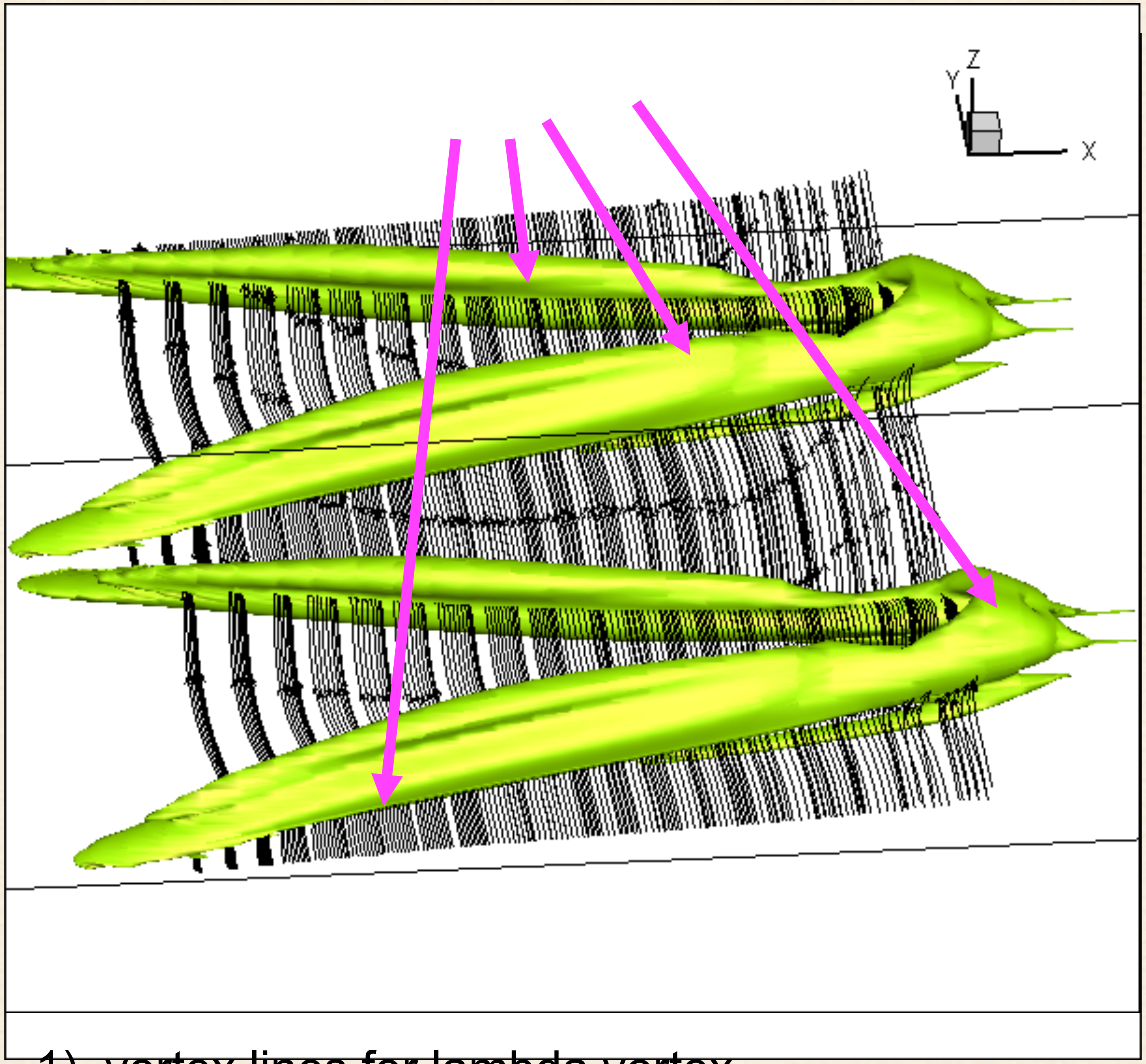
It is a serious mistake to consider vortex as vorticity tube or vortex tube (see Davidson's book)



No, vorticity line cannot penetrate vortex tube

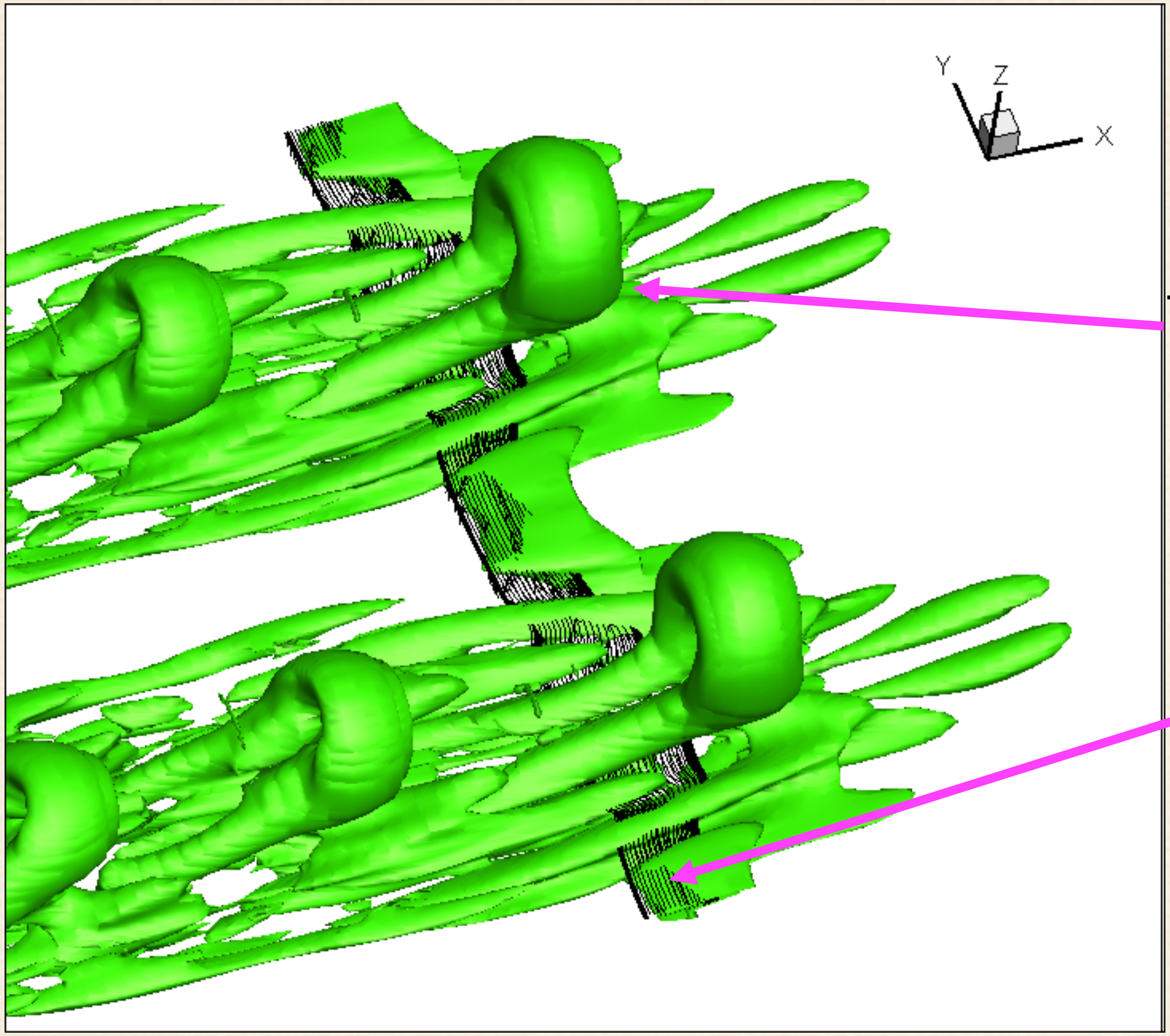
Is Lambda vortex a vortex tube?
Lambda vortex should be Lambda Rotation Cores

Lambda is a rotation center not vortex tubes as vortex filaments penetrated Lambda all times



1) vortex lines for lambda vortex
2) vortex lines for lambda vortex

First vortex ring



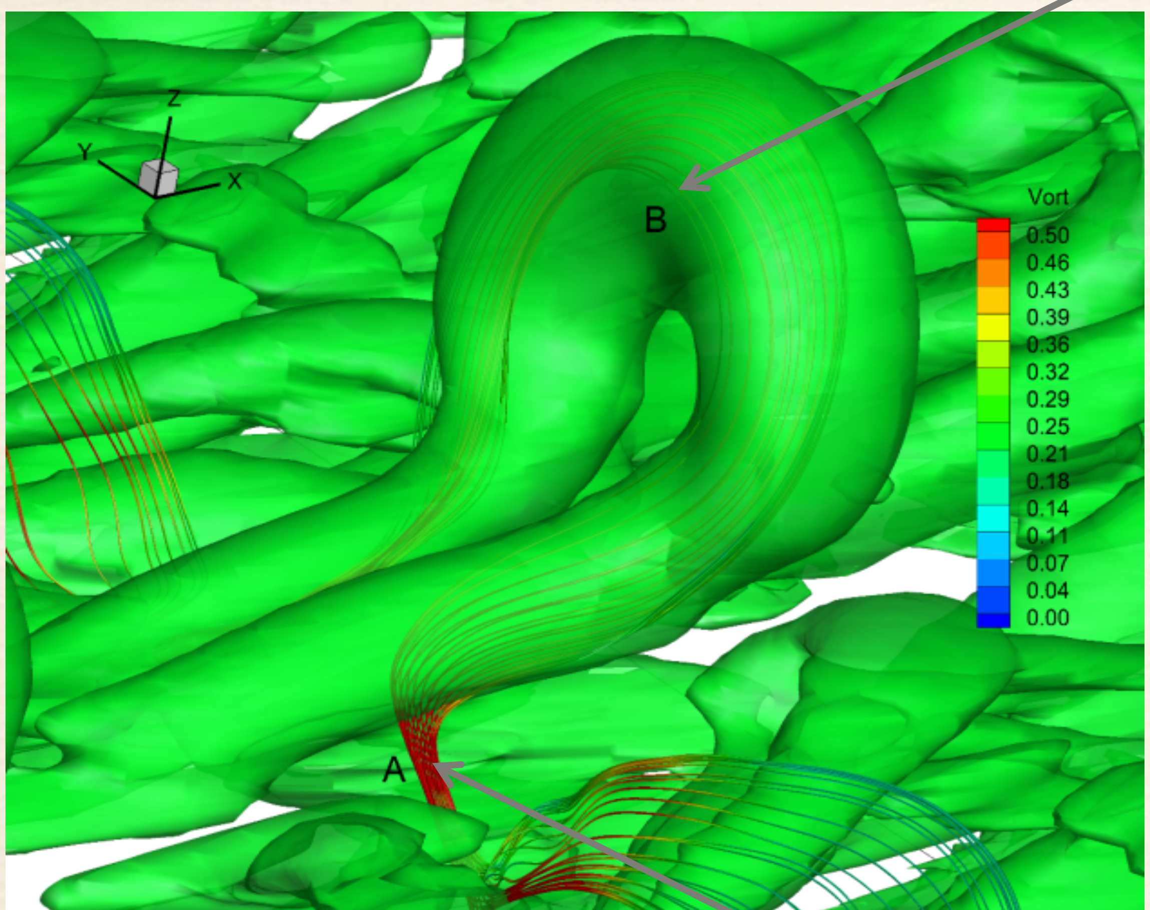
Later time steps: filaments are stretched and become longer

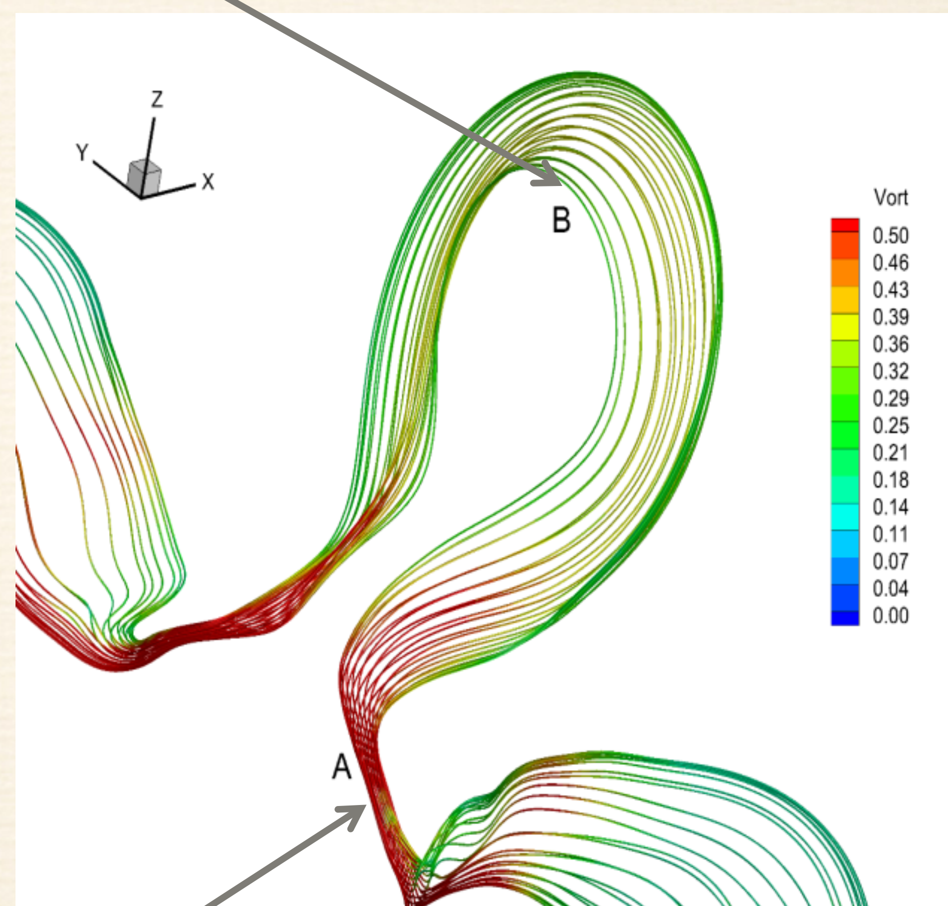
Vorticity lines from neighboring boundary layer

Late stage of the development of vortex lines for 1st ring

Is vortex a vorticity tube? No
Is vortex a congregation of vorticity lines? No
Vortex has smaller vorticity and vorticity lines dispersion

Vorticity is smaller inside vortex





Vorticity is larger outside of vortex

Is vorticity larger when vortex appears (rotation fast)

No, vorticity does not represent rotation

$|\nabla\times\vec{V}|$ =0.8, no rotation

$|\nabla\times\vec{V}|$ =0.4, rotation

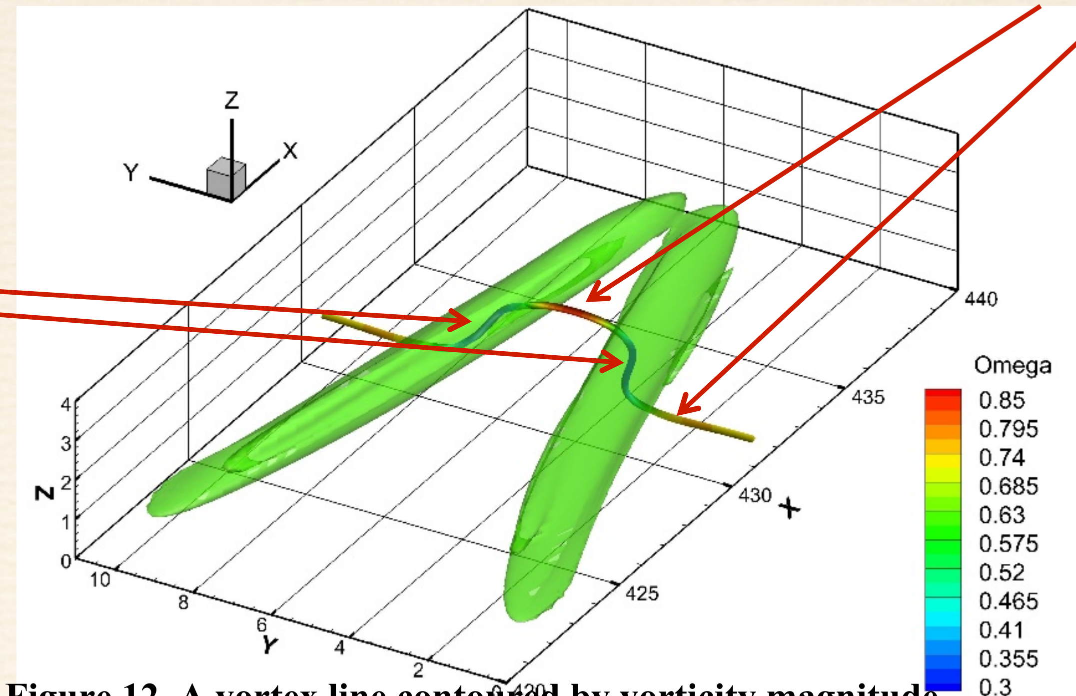


Figure 12. A vortex line contoured by vorticity magnitude

Vorticity dispersion

Vorticity congregation

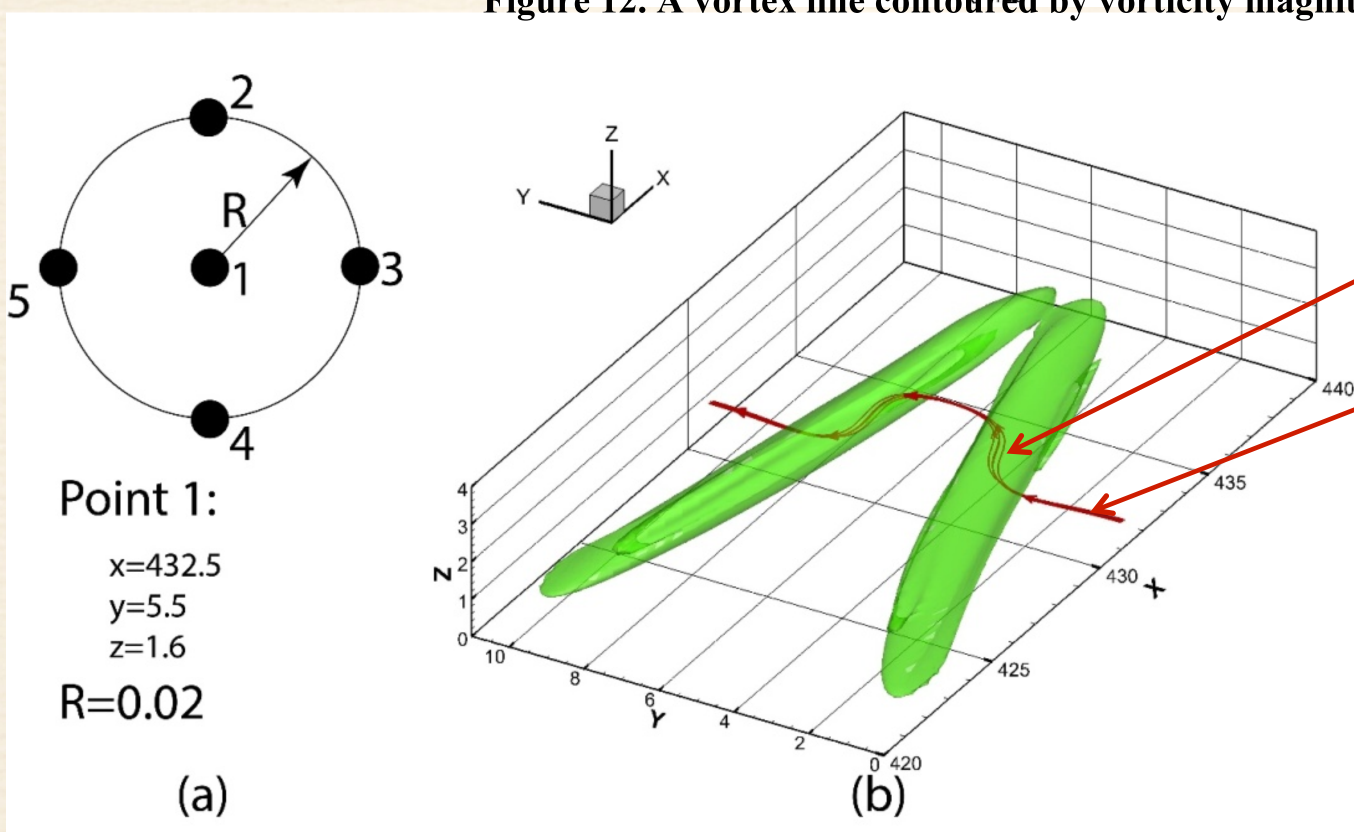


Figure 13. (a) The origination points of the five vortex filaments; (b) The Λ-vortex with the five vortex lines

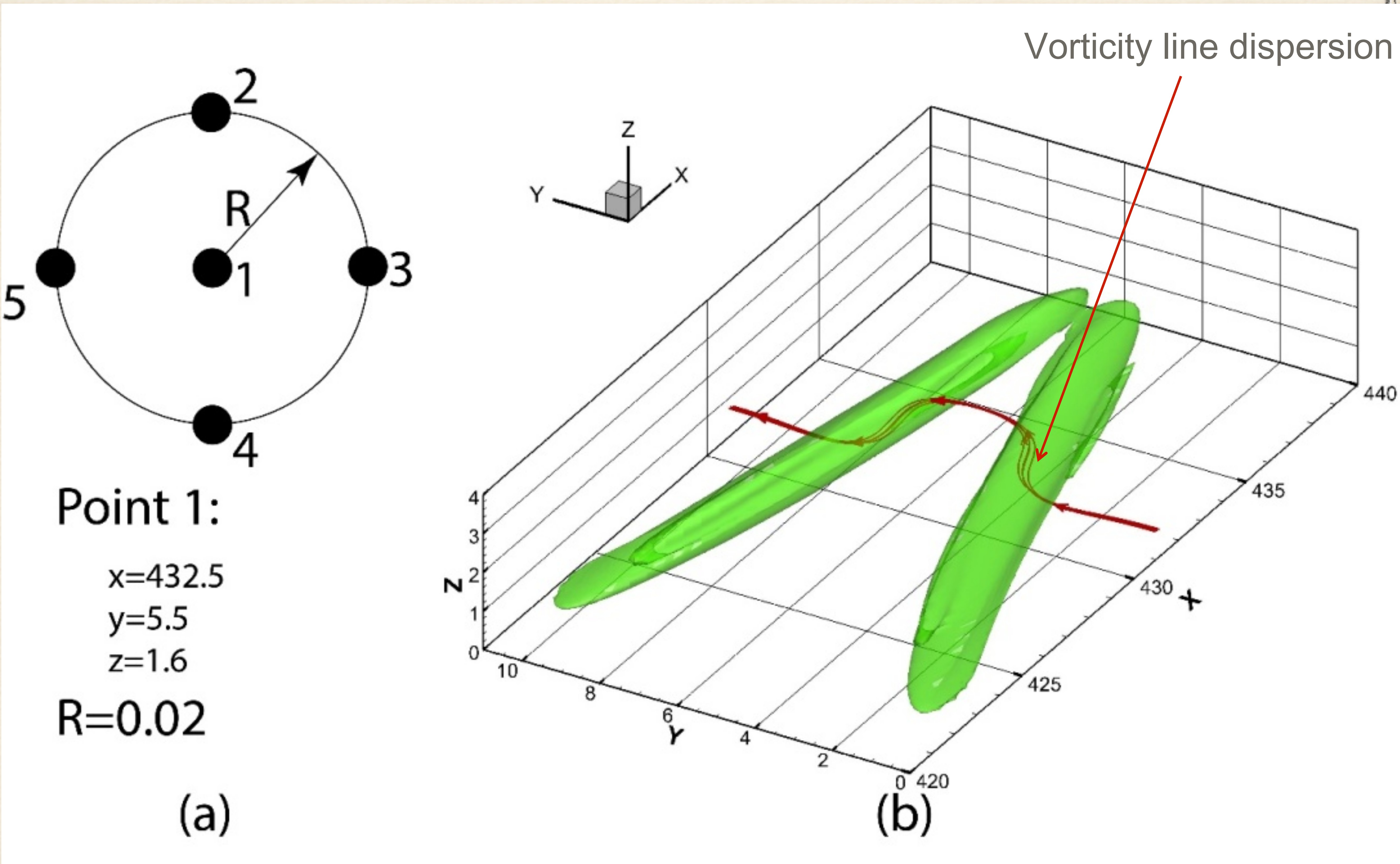


Figure 13. (a) The origination points of the five vortex filaments; (b) The Λ-vortex with the five vortex lines

12. Helmoholtz particle velocity decomposition revisit

$$\vec{V}(\vec{X}+d\vec{X}) = \vec{V}(\vec{X}) + d\vec{V}$$

$$d\vec{V} = d\vec{X} \bullet \nabla \vec{V}$$

$$\nabla \vec{V} = \frac{1}{2}(\nabla \vec{V} + \nabla \vec{V}^T) + \frac{1}{2}(\nabla \vec{V} - \nabla \vec{V}^T) = \vec{\varepsilon} + \frac{1}{2}(\nabla \vec{V} - \nabla \vec{V}^T)$$

$$d\vec{V} = d\vec{X} \bullet \vec{\varepsilon} - d\vec{X} \times \vec{\omega}, \ where\ \vec{\omega} = \frac{1}{2}\nabla \times \vec{V}$$

Decomposed to deformation and rotation – Serious misunderstanding

$$\omega = \frac{1}{2}\nabla \times \vec{V}$$

Is vorticity which does not mean rotation
(vorticity is not vortex or rotation)

$$\nabla \times \vec{V} = 0$$

Irrotational Flow

$$\nabla \times \vec{V} \neq 0$$

Rotational Flow?? No
Could still be irrotational like Blasius solution

Vorticity should be further decomposed to vortical vorticity and non-vortical vorticity

Fluid motion: Translation, Deformation, vortical Vorticity, Non-vortical Vorticity

Velocity Decomposition

Assume A represents the velocity gradient tensor:

$$A = \nabla\vec{V} = \frac{1}{2}(\nabla\vec{V} + \nabla\vec{V}^T) + \frac{1}{2}(\nabla\vec{V} - \nabla\vec{V}^T)$$

$$d\vec{V} = \nabla\vec{V}\cdot d\vec{l} = \underbrace{\overset{\leftrightarrow}{S}\cdot d\vec{l}}_{Deformation} + \underbrace{\nabla\times\vec{V}\times d\vec{l}}_{Vorticity}$$

$$\begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{bmatrix} = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{1}{2}(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}) & \frac{1}{2}(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}) \\ \frac{1}{2}(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}) & \frac{\partial v}{\partial y} & \frac{1}{2}(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}) \\ \frac{1}{2}(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}) & \frac{1}{2}(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}) & \frac{\partial w}{\partial z} \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{2}(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}) & \frac{1}{2}(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}) \\ -\frac{1}{2}(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}) & 0 & \frac{1}{2}(\frac{\partial v}{\partial z} - \frac{\partial w}{\partial y}) \\ -\frac{1}{2}(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}) & -\frac{1}{2}(\frac{\partial v}{\partial z} - \frac{\partial w}{\partial y}) & 0 \end{bmatrix}$$

$$= \begin{bmatrix} S_{xx} & S_{xy} & S_{xz} \\ S_{yx} & S_{yy} & S_{yz} \\ S_{zx} & S_{zy} & S_{zz} \end{bmatrix} + \begin{bmatrix} 0 & -\frac{1}{2}\omega_z & \frac{1}{2}\omega_y \\ \frac{1}{2}\omega_z & 0 & -\frac{1}{2}\omega_x \\ -\frac{1}{2}\omega_y & \frac{1}{2}\omega_x & 0 \end{bmatrix} = S_{ij} - \frac{1}{2}\varepsilon_{ijk}\omega_k = S_{ij} + W_{ij}$$

$$where\ i, j, k = 1, 2, 3;\ \varepsilon_{ijk}: \quad \varepsilon_{111} = \varepsilon_{222} = \varepsilon_{333} = 0; \varepsilon_{123} = \varepsilon_{231} = \varepsilon_{312} = 1;$$

$$\varepsilon_{132} = \varepsilon_{213} = \varepsilon_{321} = -1; or\ \varepsilon_{ijk} = \begin{cases} 0 & i = j \\ 1 & j = i+1\ or\ 4-i \\ -1 & j = i-1\ or\ i+2 \end{cases}$$

Vorticity does not mean rotation and should be further decomposed to vortical vorticity and non-vortical vorticity

$$\nabla \times \vec{V} = \vec{R} + (\nabla \times \vec{V} - \vec{R})$$

$$\nabla \times \vec{V} = \Omega \frac{\vec{R}}{|\vec{R}|} + (\nabla \times \vec{V} - \Omega \frac{\vec{R}}{|\vec{R}|})$$

$\Omega \frac{\vec{R}}{|\vec{R}|}$ represents vortical vorticity.

$(\nabla \times \vec{V} - \Omega \frac{\vec{R}}{|\vec{R}|})$ represents vortical vorticity.

How to find Ω

How to find Ω

$$\nabla\vec{V} = \frac{1}{2}\left(\nabla\vec{V} + \nabla\vec{V}^T\right) + \frac{1}{2}\left(\nabla\vec{V} - \nabla\vec{V}^T\right) = A + B$$

Symmetric Anti-symmetric

$$a = trace(A^T A) = \sum_{i=1}^{3}\sum_{j=1}^{3}(A_{ij})^2$$

$$b = trace(B^T B) = \sum_{i=1}^{3}\sum_{j=1}^{3}(B_{ij})^2$$

$$\Omega = \frac{b}{a+b+\epsilon} \qquad \varepsilon = 1.0\times 10^{-3} \qquad 0 < \Omega < 1$$

$For\ rigid\ rotation, a = 0$, and then $\Omega = 1$

$For\ pure\ deformation, b = 0$, and then $\Omega = 0$

Our numerical tests show $\Omega = 0.52$ $represants\ vortex\ boundary$

Note that we must use non-dimensional length and velocity to calculate Ω !!

How to Define Vortex?

There is no mathematical definition for vortex

We try to give mathematical definition:

Vortex is a region where vorticity overtakes deformation or $\Omega > 0.5$

We pick $\Omega = 0.52$ to show the vortex surface

No matter how big or how small of vorticity,
if deformation overtakes vorticity, there is no vortex
Vorticity magnitude is not directly related to VORTEX

Comments on Existing Vortex Identification Methods

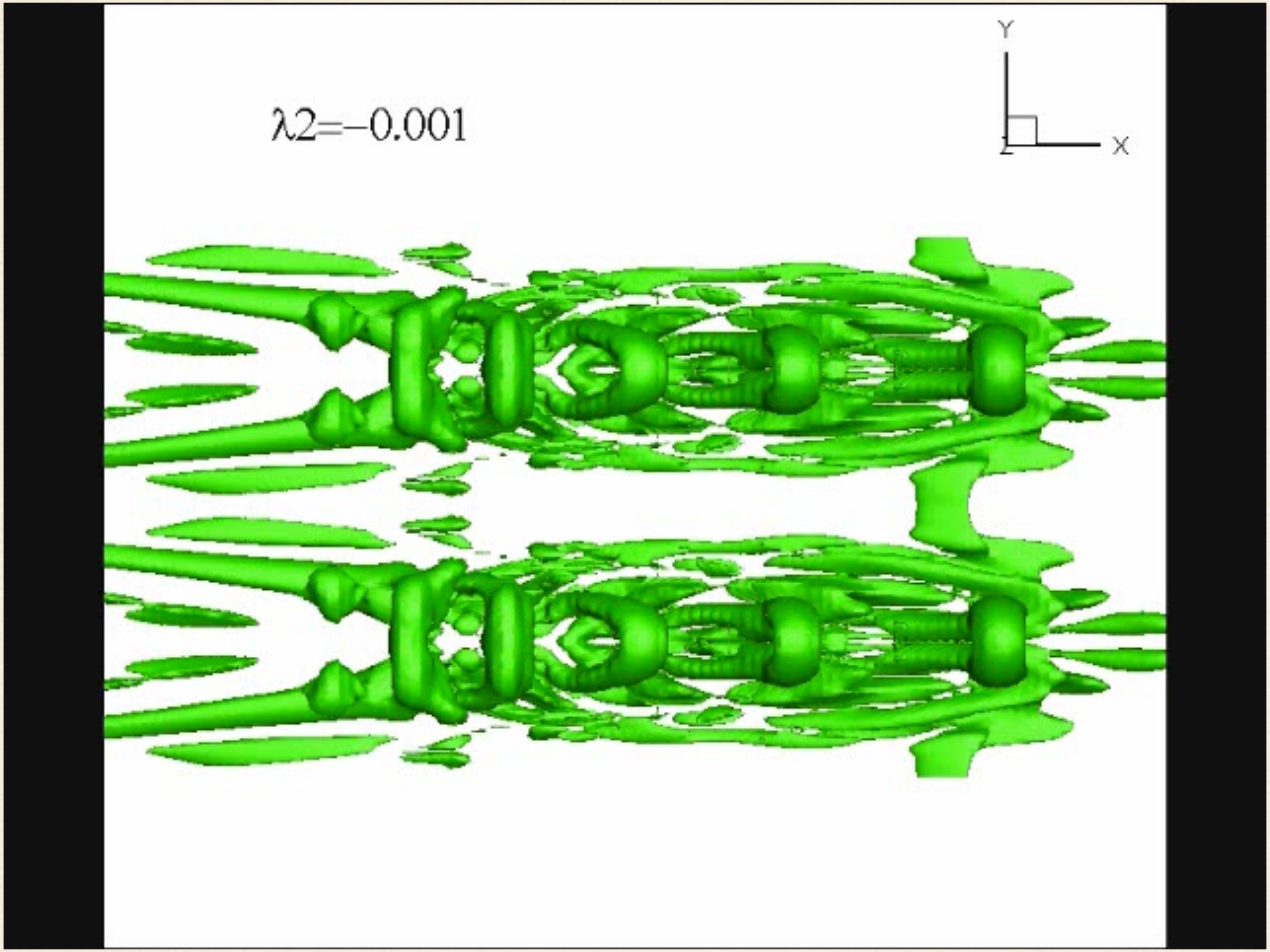
All current vortex identification methods have following disadvantages:

1. Require a proper threshold which is different case by case, time by time and even area by area.

2. Different threshold will give different vortex structures – No one knows which threshold is proper or not proper and no one knows which vortex structure is "correct vortex structure" or "incorrect vortex structure"

3. Cannot capture both strong vortex and weak vortex- Either capture strong vortex , but miss weak vortex or capture weak vortex but smear strong vortex

4. Did not give mathematical definition for vortex


λ2=−0.001
Y
X

New Omega Vortex Identification Method

1. Give the mathematical definition for vortex Ω>0.5
2. Easy to perform:

$$\nabla \vec{V} = \frac{1}{2}(\nabla \vec{V} + \nabla \vec{V}^T) + \frac{1}{2}(\nabla \vec{V} - \nabla \vec{V}^T) = A + B$$

$$\Omega = \frac{\| B^T B \|_F}{\| A^T A \|_F + \| B^T B \|_F + \varepsilon}$$

3. Physical meaning is clear: vorticity overtakes deformation
3. No threshold is needed
4. Able to capture both strong and weak vortices
5. Suggestions: Use the new Omega vortex identification method to show vortex structure in flow field

Reference Paper:

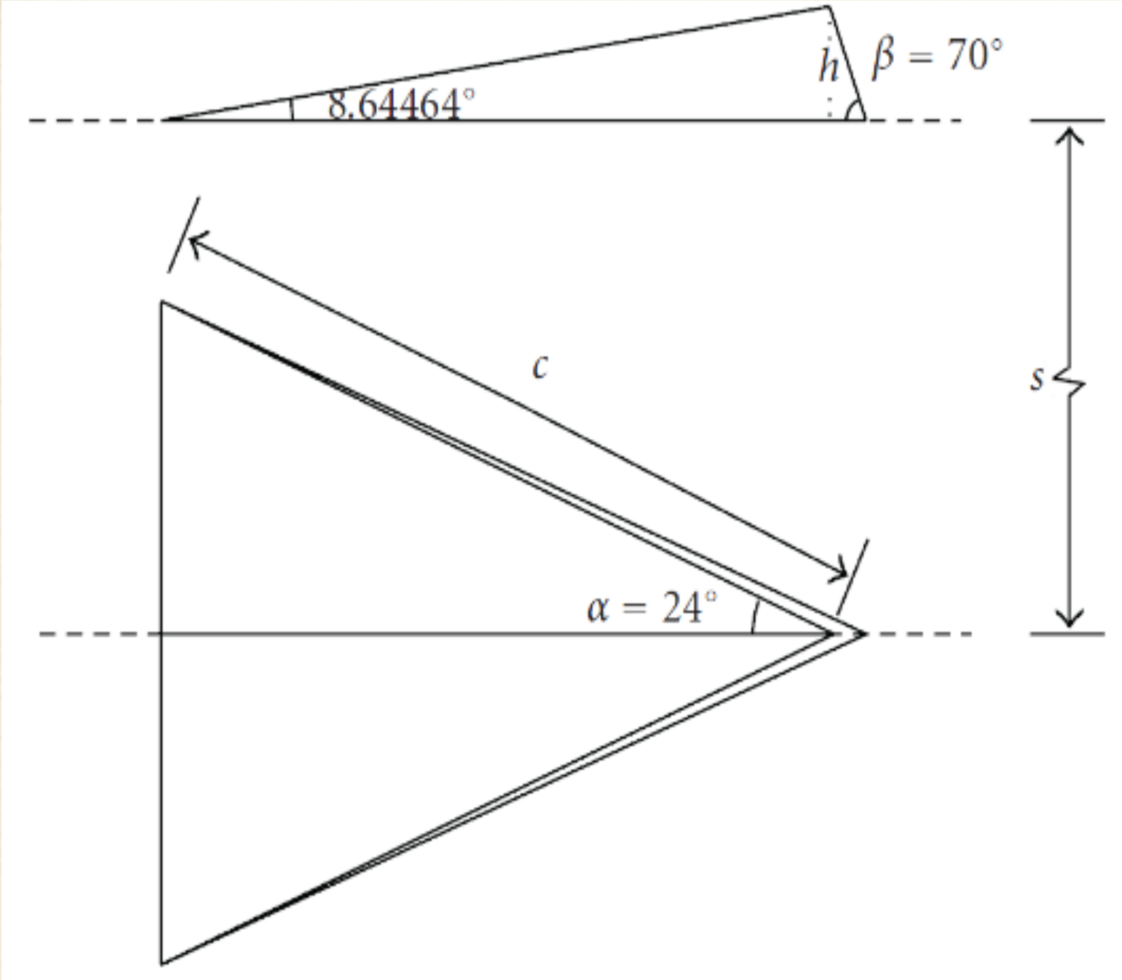
LIU Chaoqun, WANG Yiqian, YANG Yong & DUAN Zhiwei, New omega vortex identification method, *Science China: Physics, Mechanics & Astronomy*, 2016, 59(8): 684711 (2016)

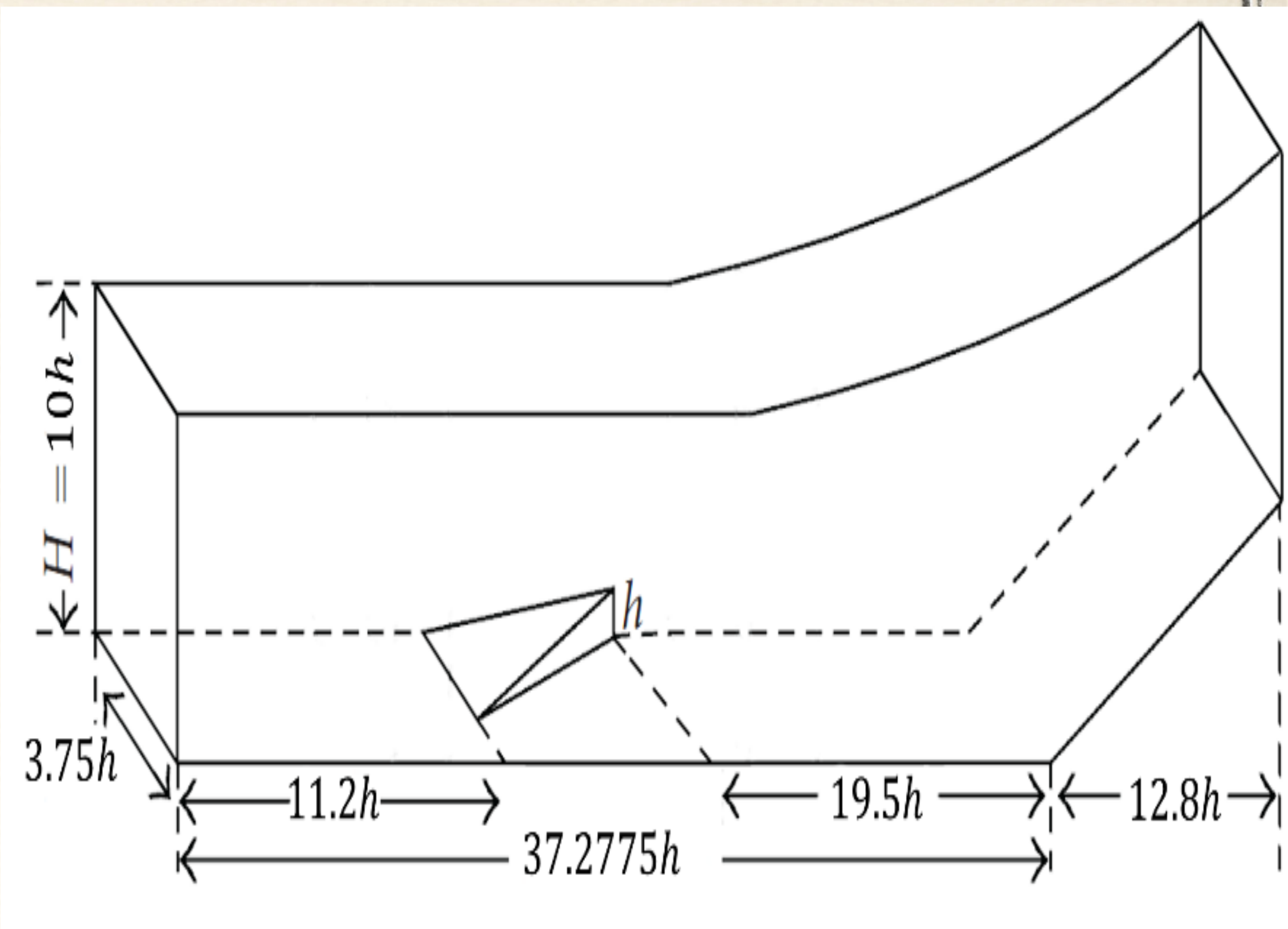
Invited Paper, Science China Press http://www.scichina.com/

Web sites:

http://www.eurekalert.org/pub_releases/2016-05/scp-anv050316.php

http://www.sciencenewsline.com/news/2016050408030039.html





(a) (b)

Fig. 4. The dimensions of (a) MVG and (b) the half domain

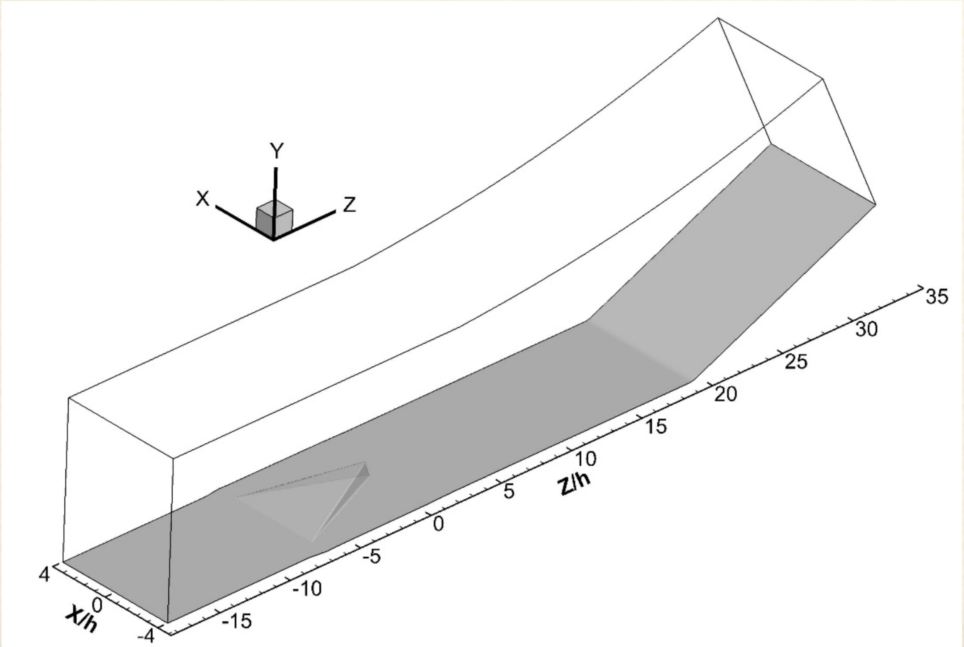


Fig. 5. The whole computational domain with x and z coordinates.

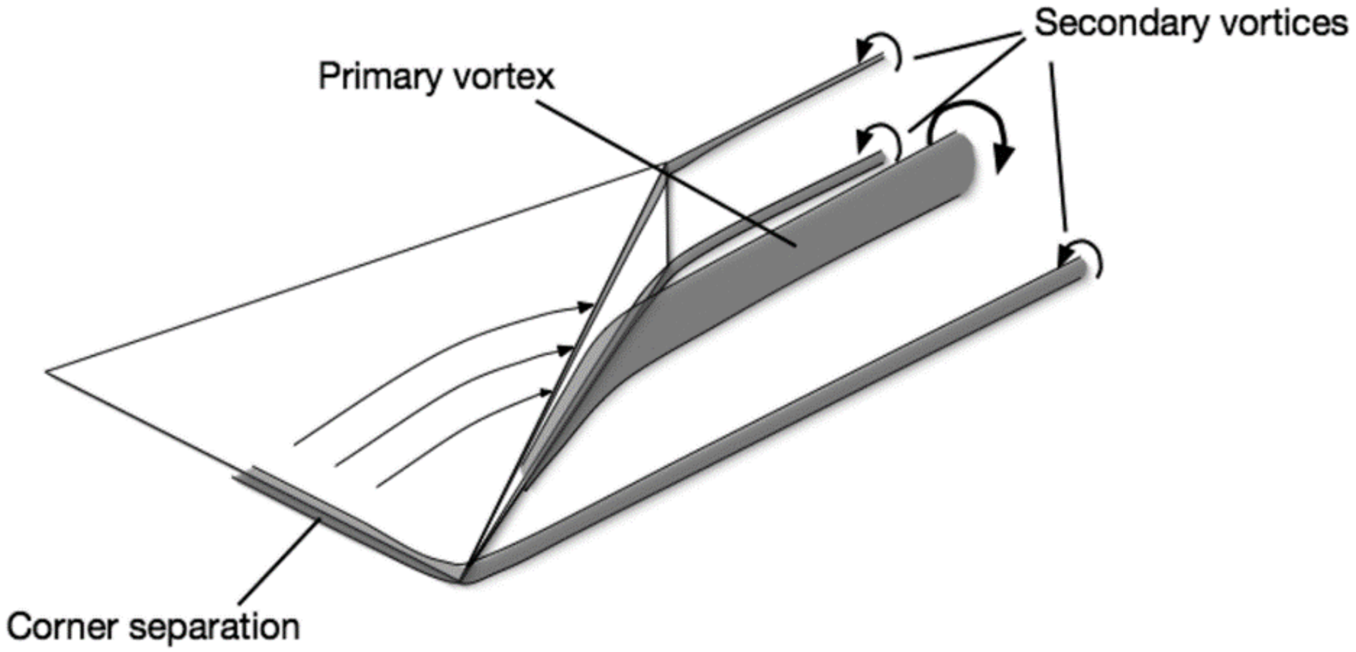


Fig. 1. Sketch of main flow features (one side only for clarity), Babinsky et al (2009) –Time averaged view

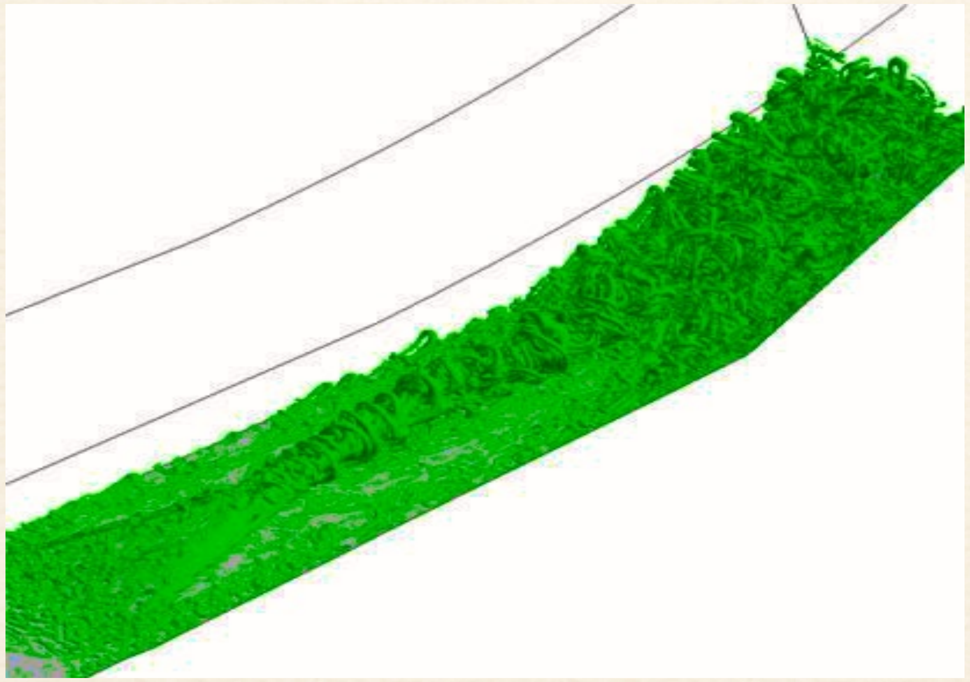


Fig. 2. Vortex ring generation by MVG due to K-H instability ($M=2.5$, $Re_{\theta}=1440$), Li and Liu (2010) –Instantaneous view

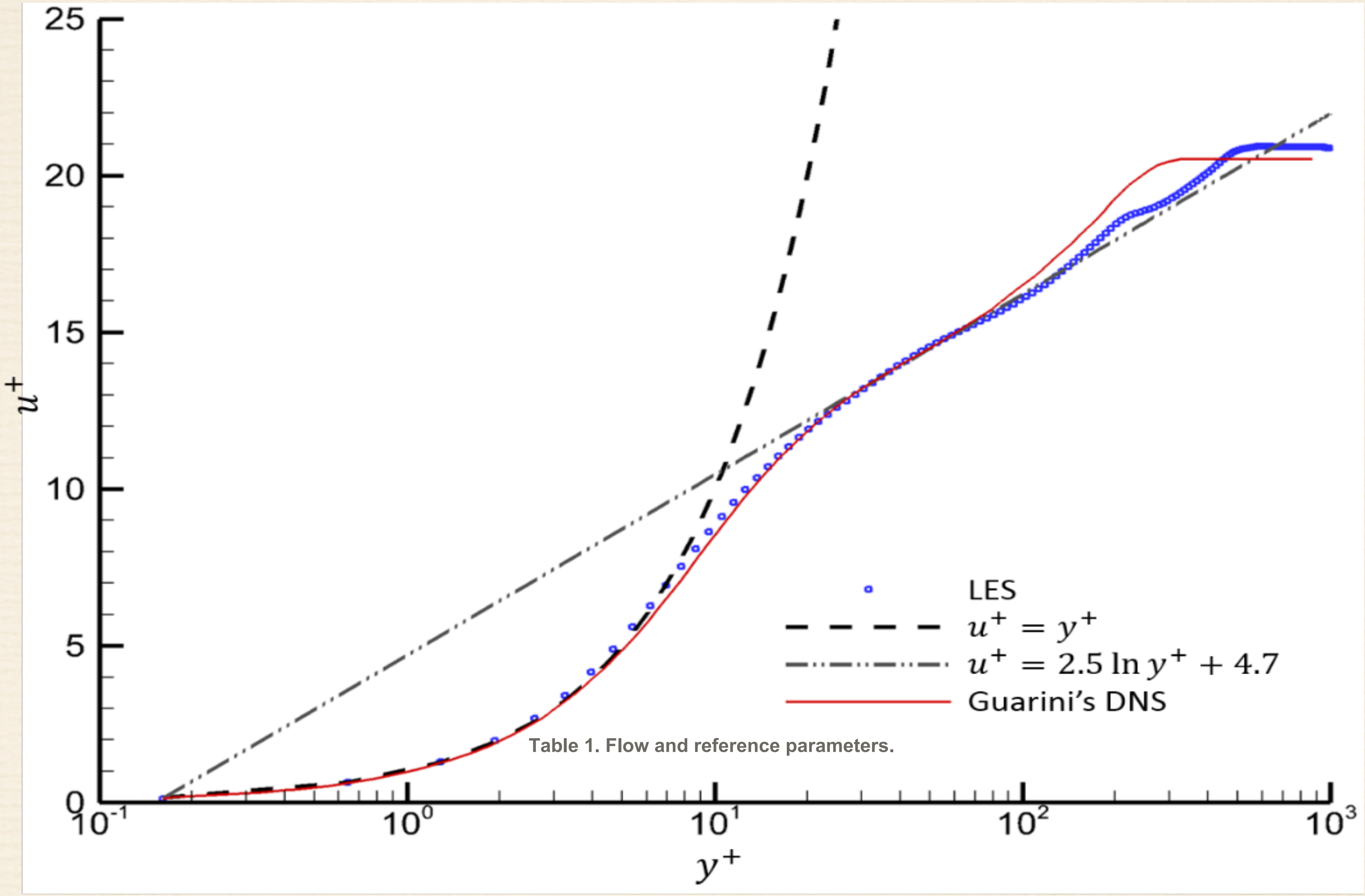
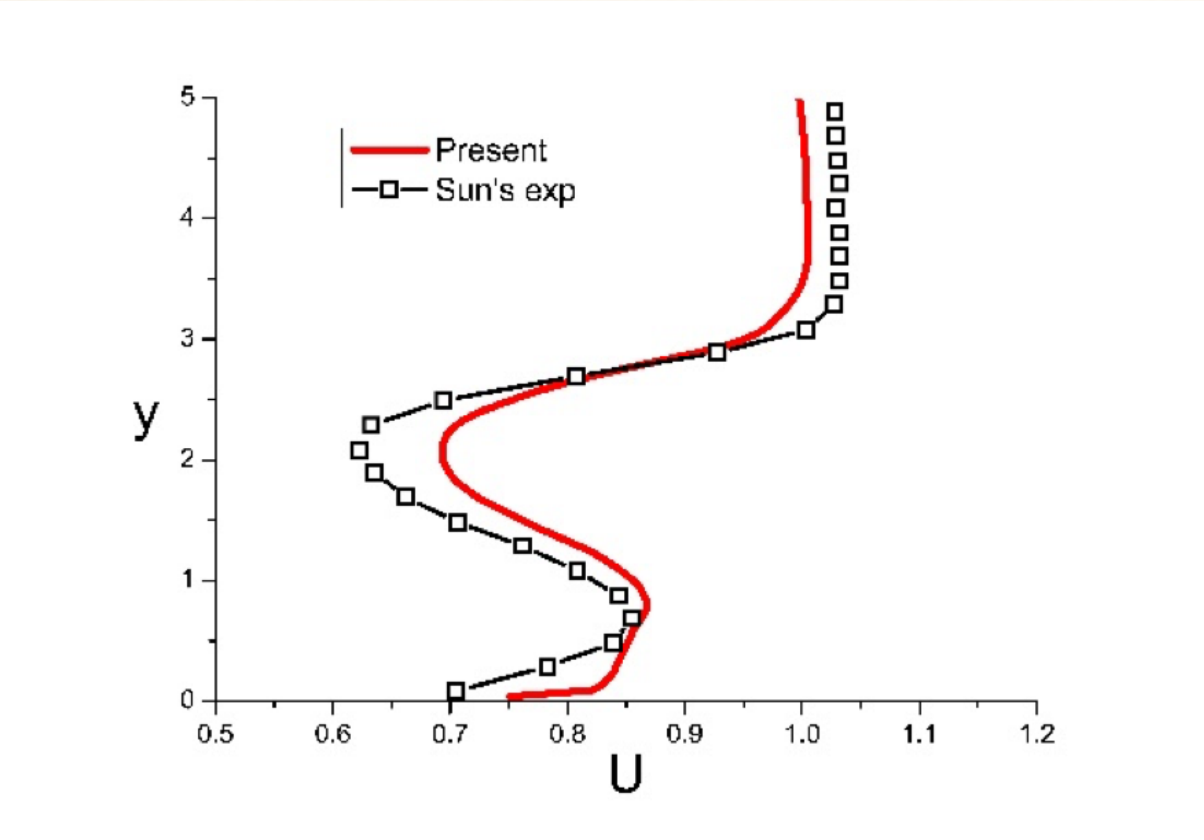


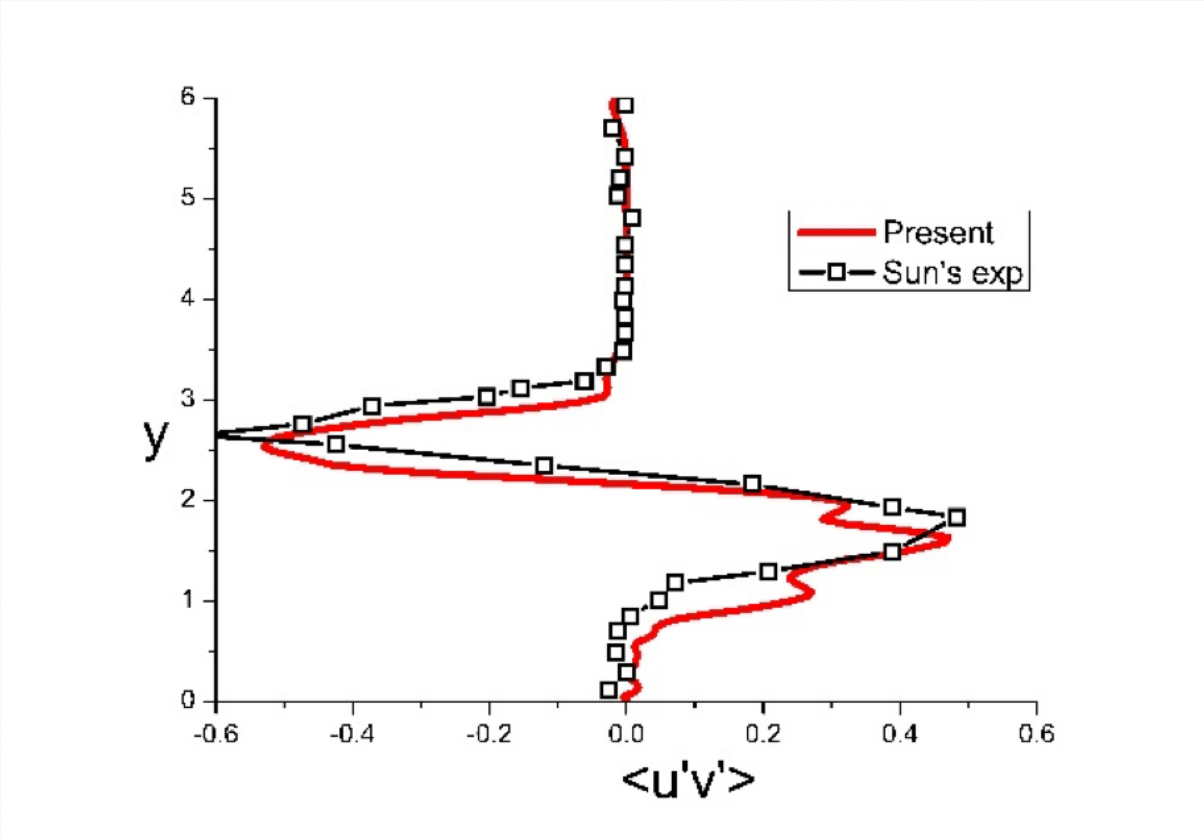
Fig. 6. Inflow boundary-layer profile at $\boldsymbol{x=0, z=-8h}$ comparison with DNS result of Guarini et al.

M_∞	Re_θ	T_∞	T_w	h	C	T
2.5	5760	288.15 K	300 K	4 mm	340 m/s	1.176×10^{-5} s

Mean Profile in MVG wake
in comparison with experiment by Delft Univ. of Technology



Mean Flow



Reynolds Stress

x/h=12.0

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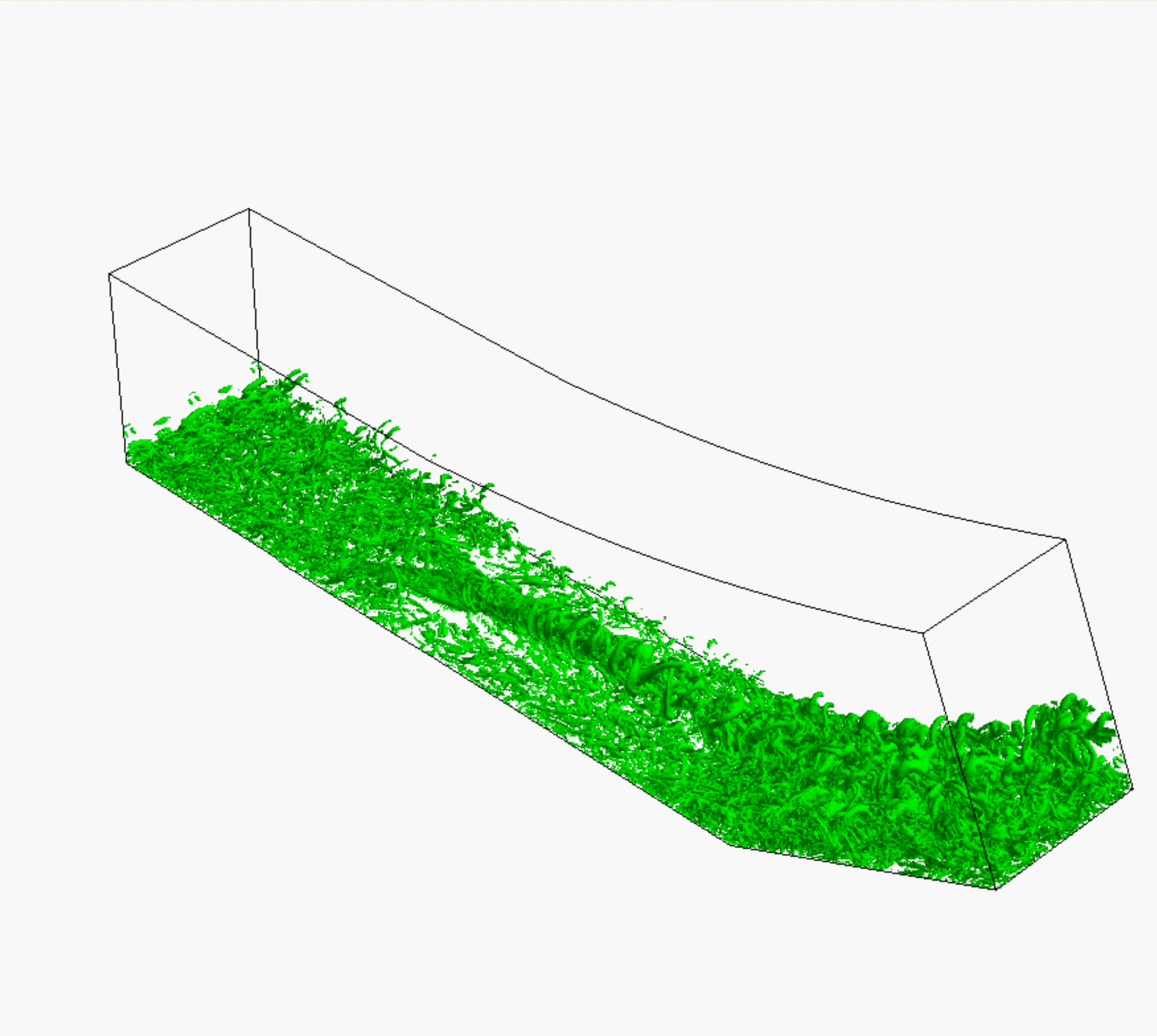
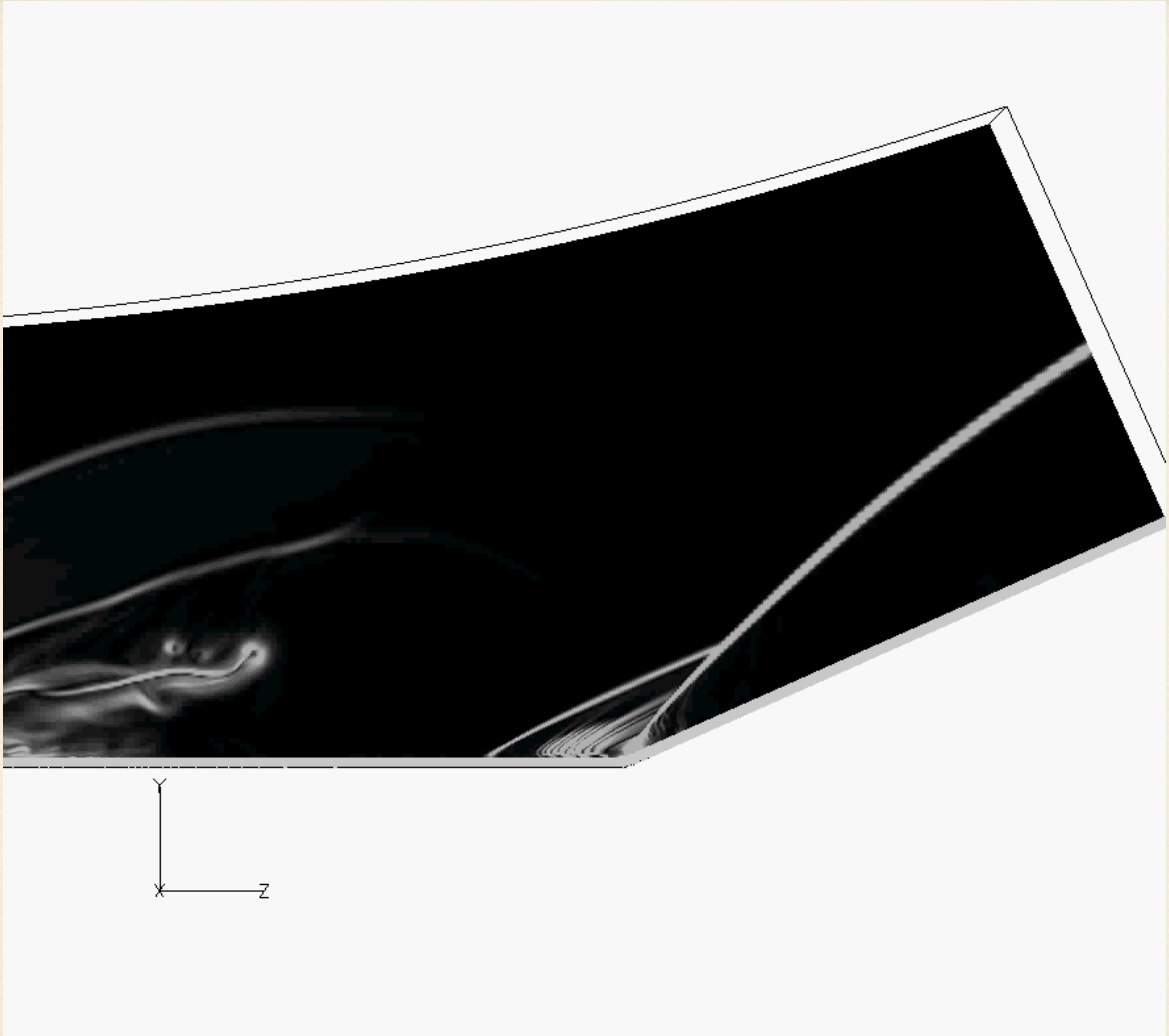
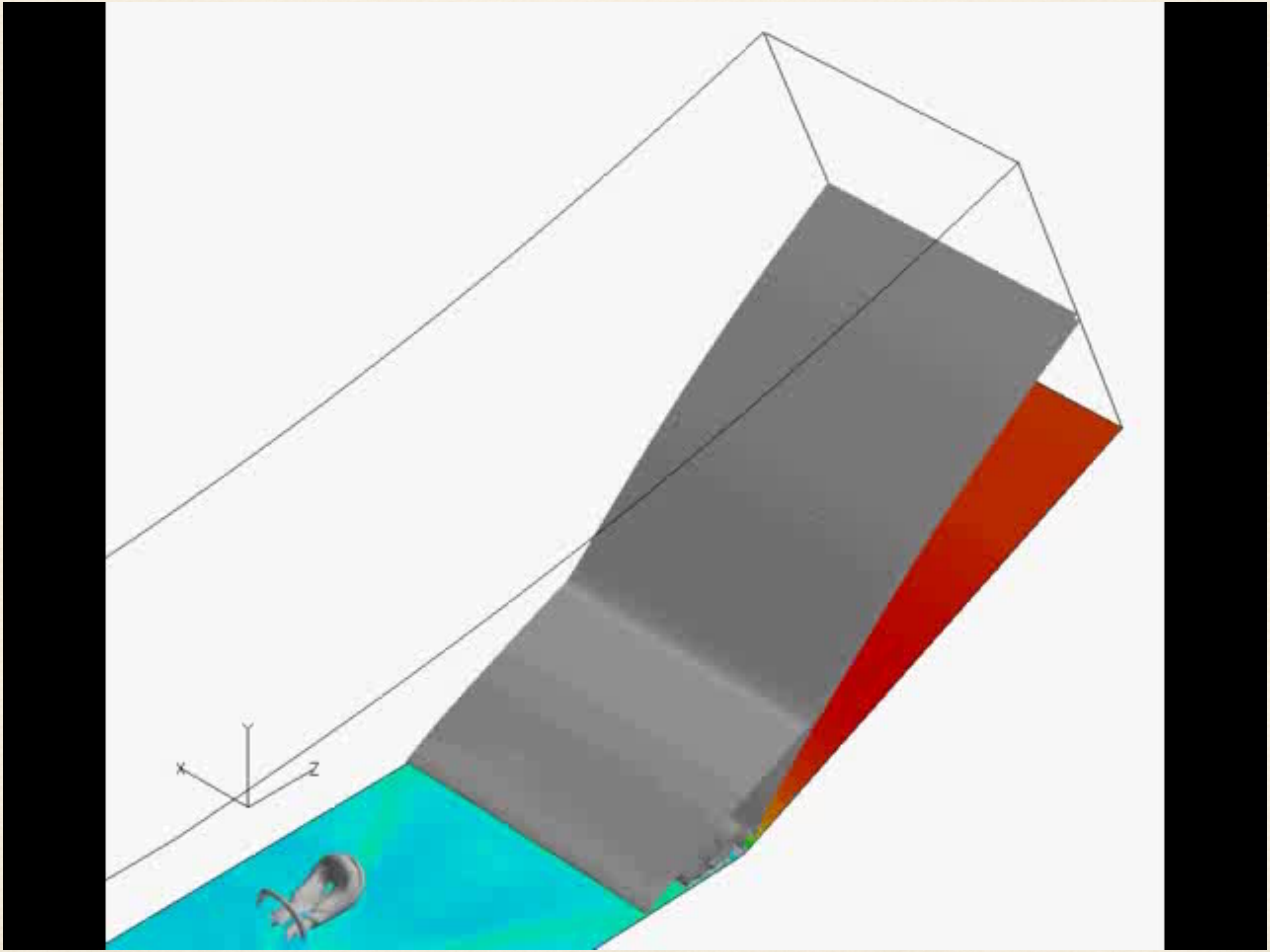
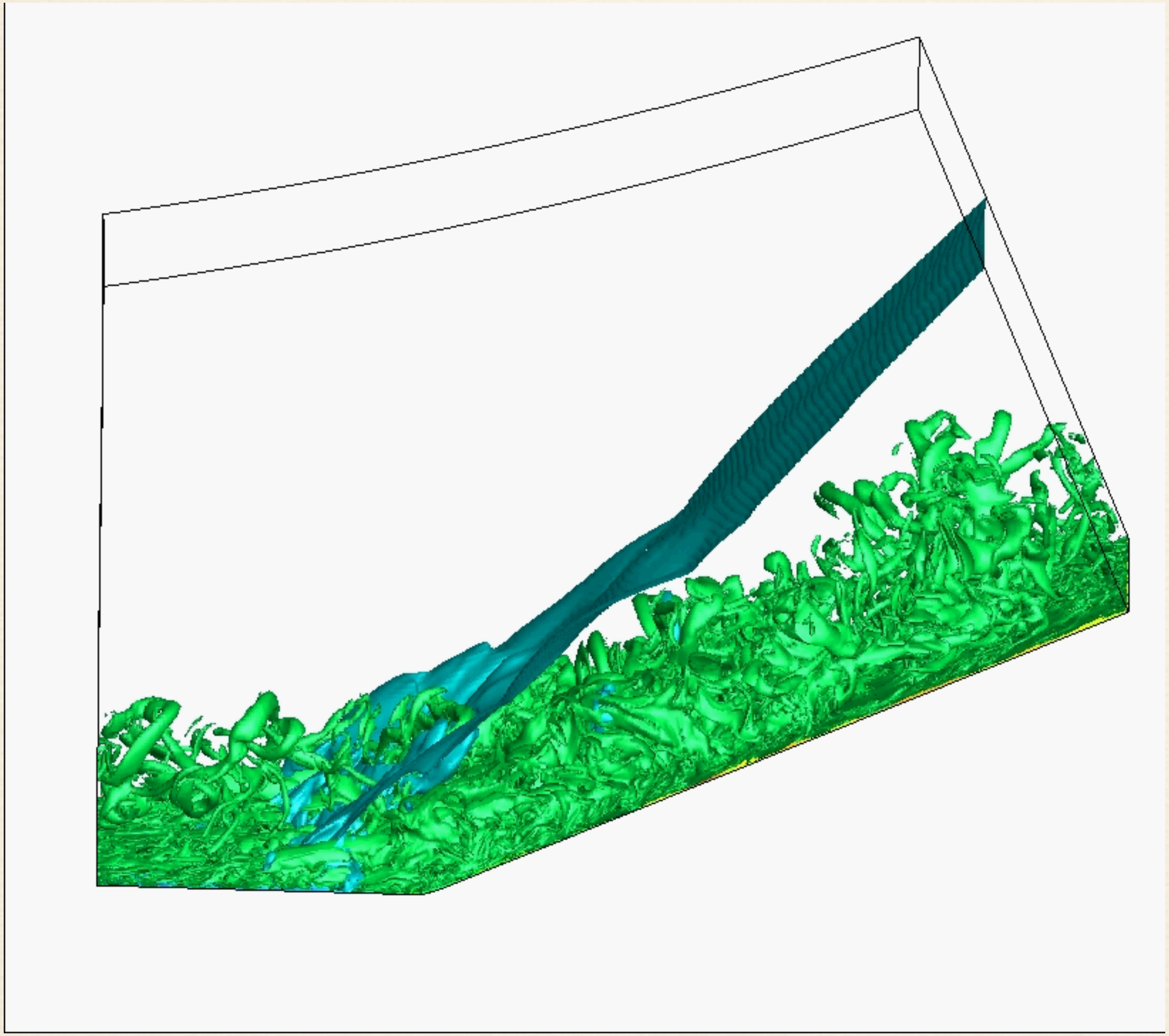
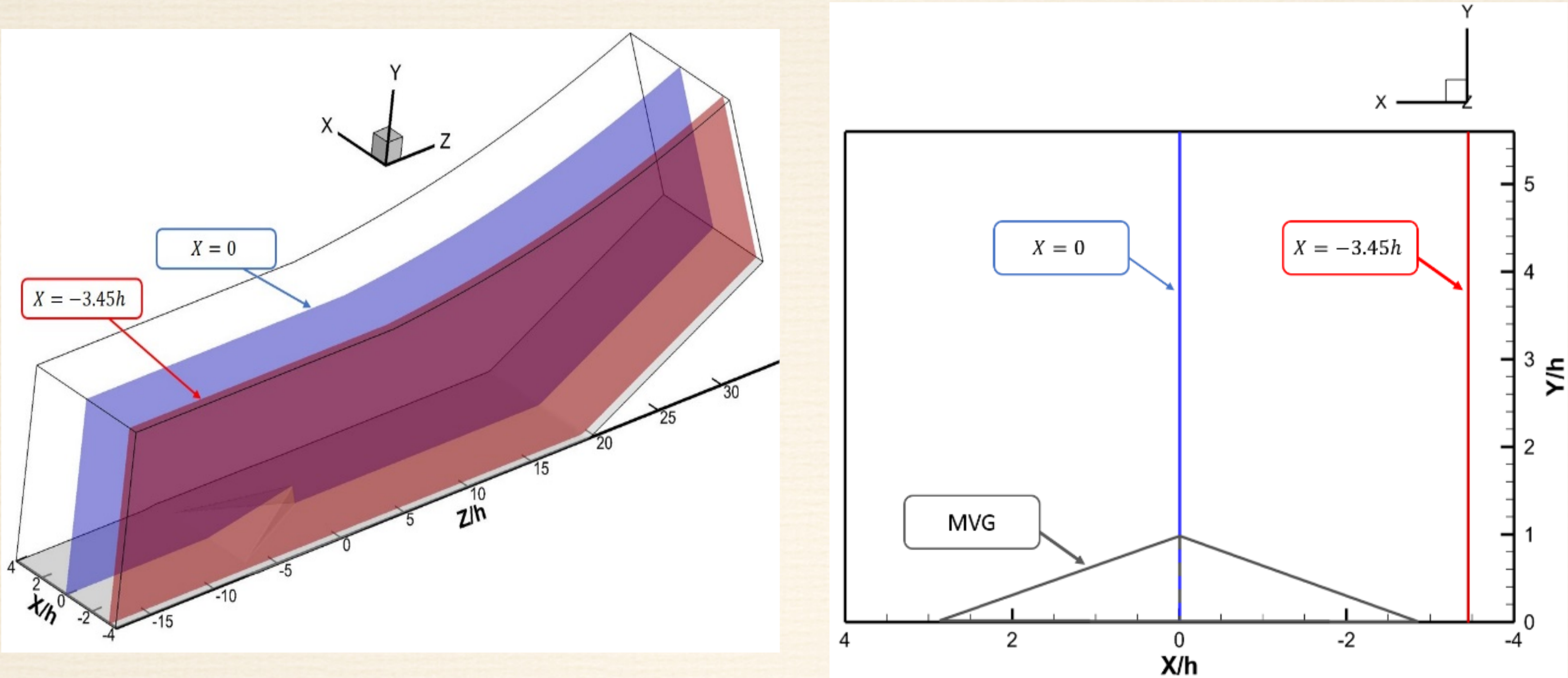
- Case4 : The theoretical study about the formation of the streamwise vortices and the vortex rings λ_2 <u>from the beginning</u>




Y
X
Z

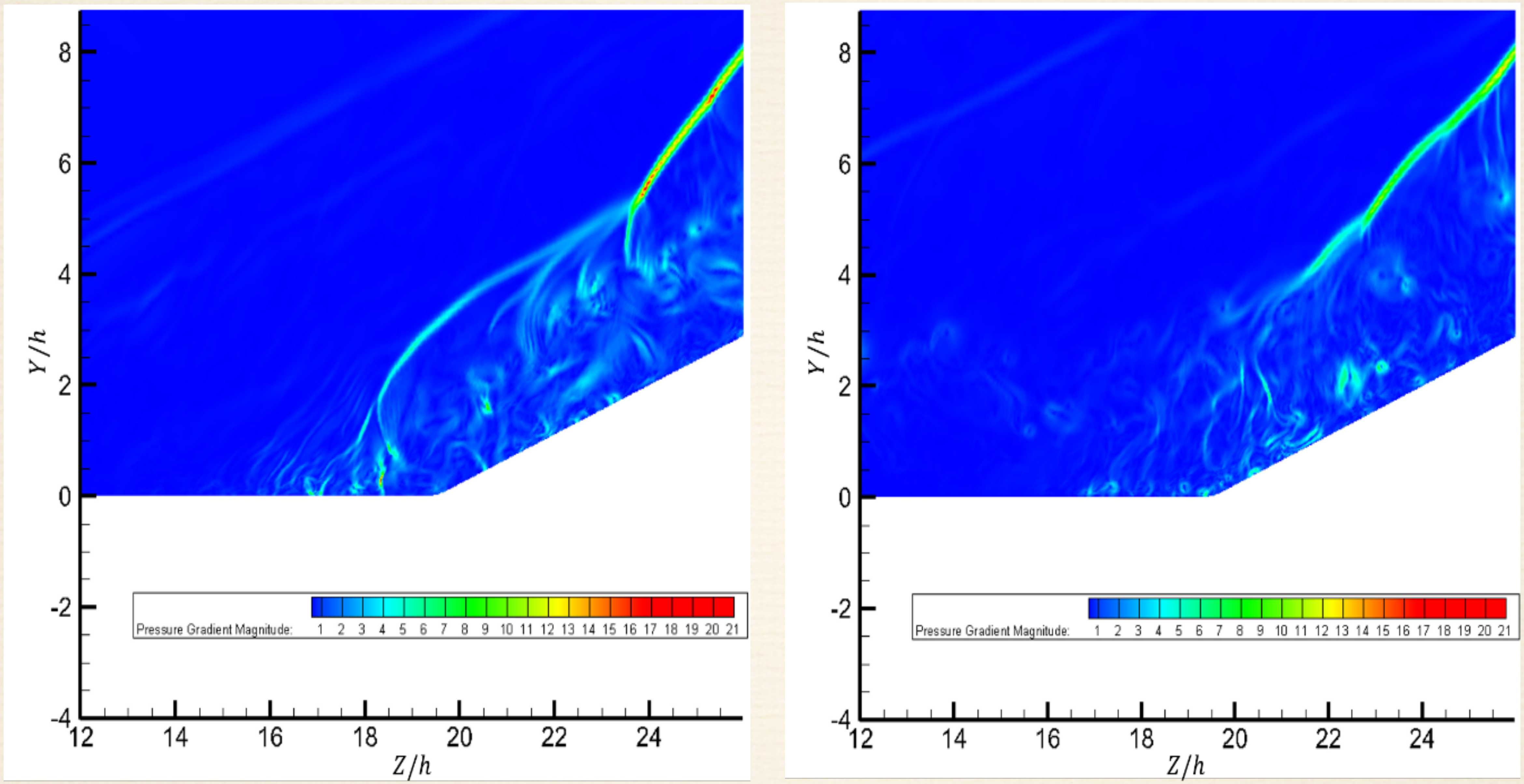

Y
X
Z





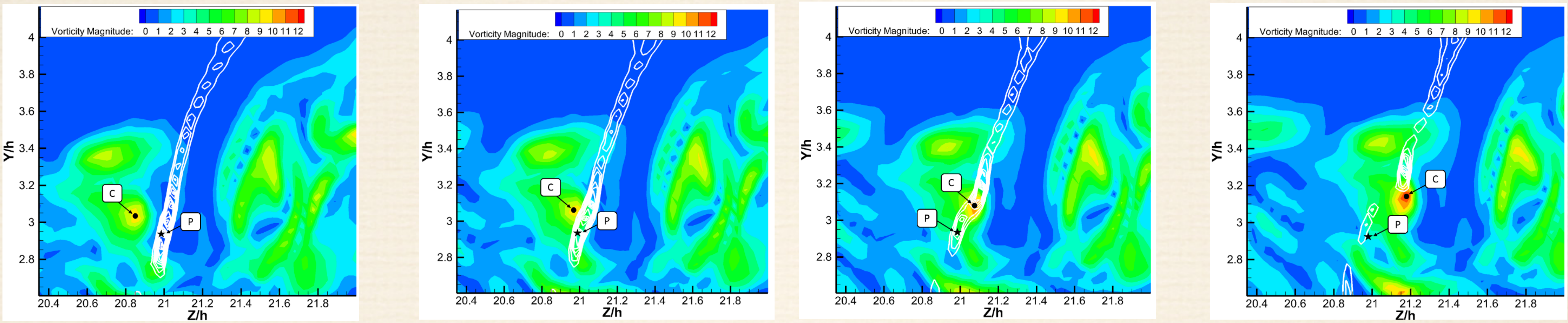
(a) (b)

Fig. 11. Position of two planes: (a) global view; (b) *XY* view

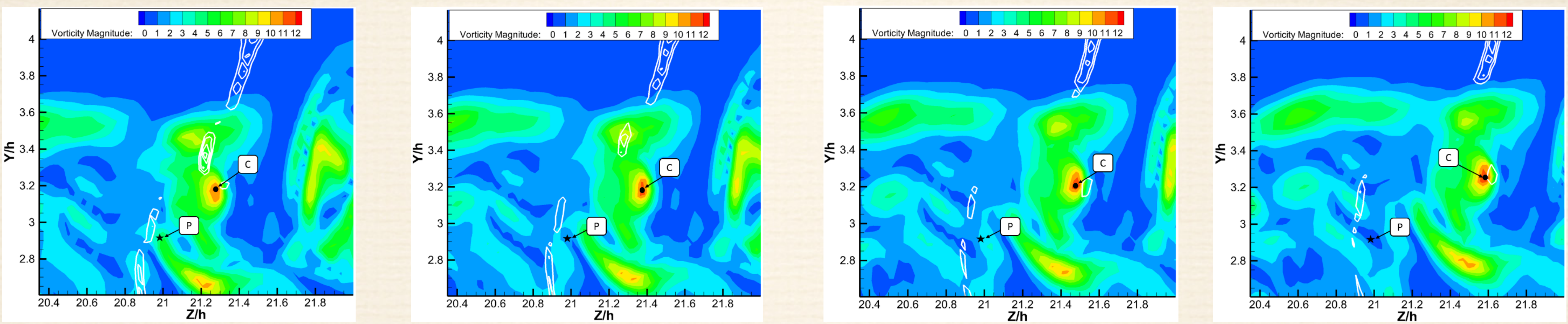


(a) $X = -3.45h$ (b) $X = 0$

Fig. 12. $\|\nabla p\|$ distribution on two planes at $t = 1800T$



(a) t=1838T (b) t=1840T (c) t=1842T (d) t=1844T



(e) t=1846T (f) t=1848T (g) t=1850T (h) t=1852T

Fig. 14. Vorticity magnitude $||\omega||$ distribution (colored contour) and pressure gradient magnitude $||\nabla p||$ (white contour lines) on the central plane $X=0$ at 8 sub-sequential time steps.

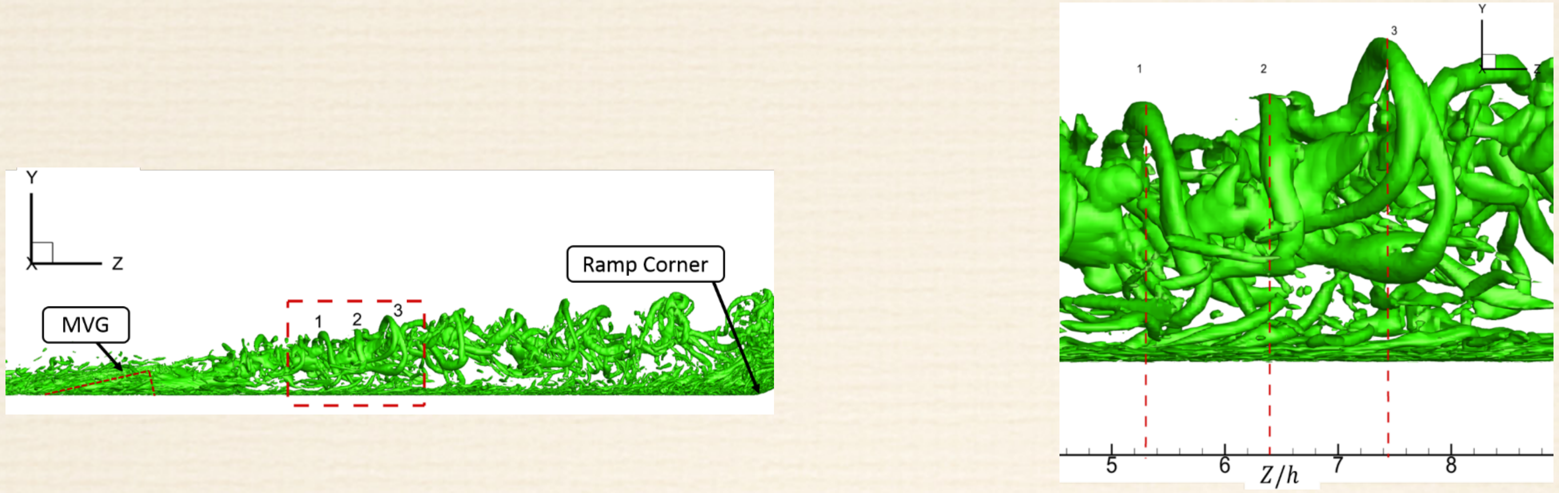


Fig. 24. Positions of three vortex rings: (a) global view; (b) local view

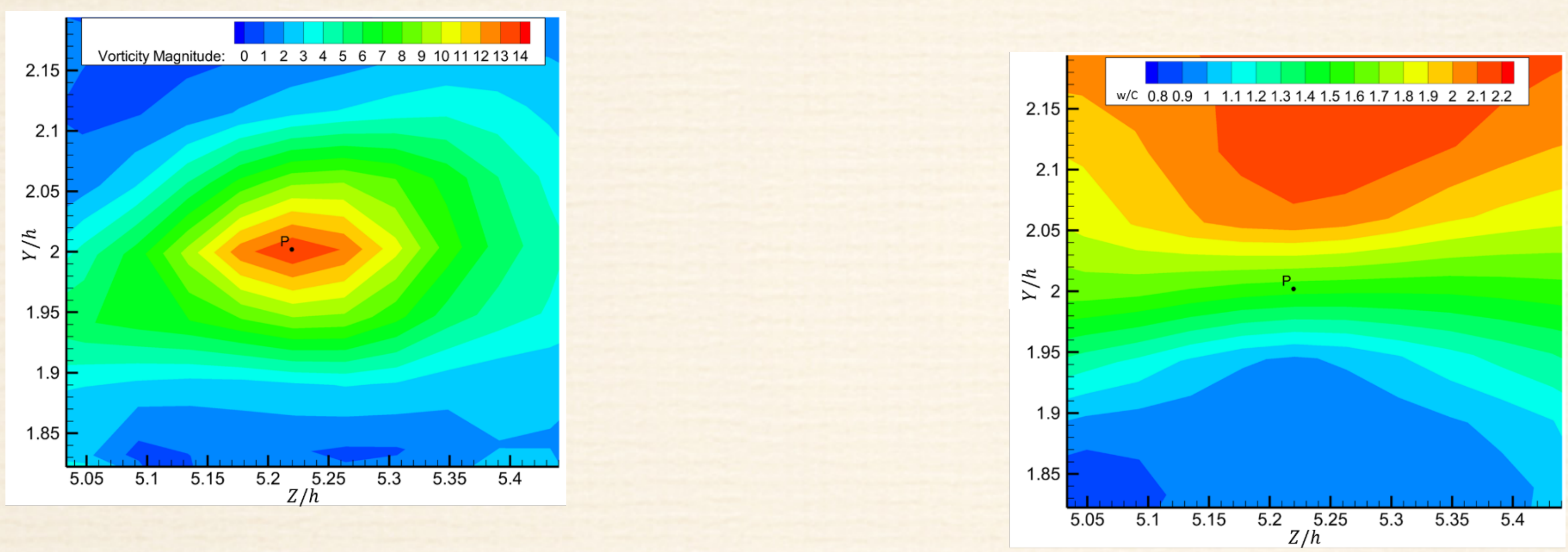


Fig. 25. Vorticity and *w*-velocity distribution around first ring head

$$V=\begin{pmatrix}\partial u/\partial x & \partial u/\partial y & \partial u/\partial z\\ \partial v/\partial x & \partial v/\partial y & \partial v/\partial z\\ \partial w/\partial x & \partial w/\partial y & \partial w/\partial z\end{pmatrix}=\begin{pmatrix}-0.6121 & 2.6205 & 0.8943\\ 1.3580 & 0.3605 & \dots\end{pmatrix}$$

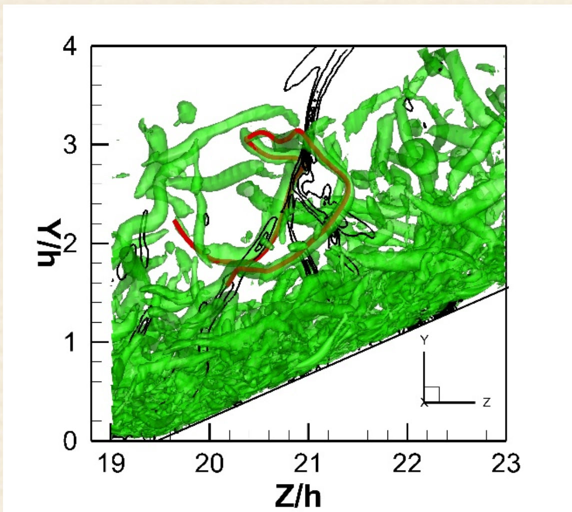
$\Omega=0.95.$

$\varpi_{vor}=9.4824.$

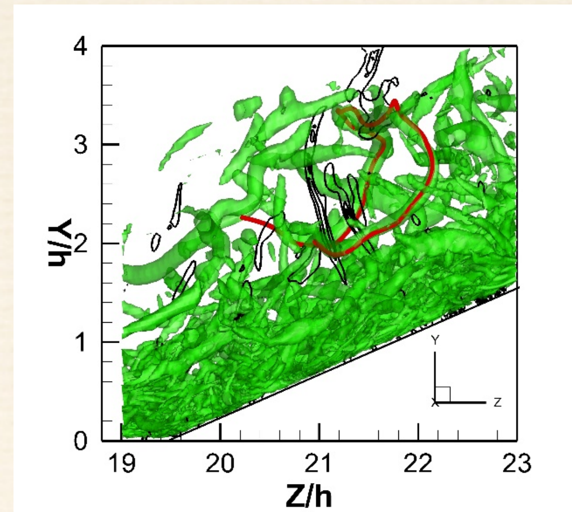
$f=w_{ring}/\Delta s=119000\ s^{-1}$

$St=fh/w_{\infty}=119000\times 0.004/2.5\times 340=0.56,$

Fast Rotation: around 64,000 circles/per second



(a) t=1838T



(b) t=1852T

Fig. 19. Position of the same ring-like vortex at two different time steps. Red circle indicates the position of ring head.

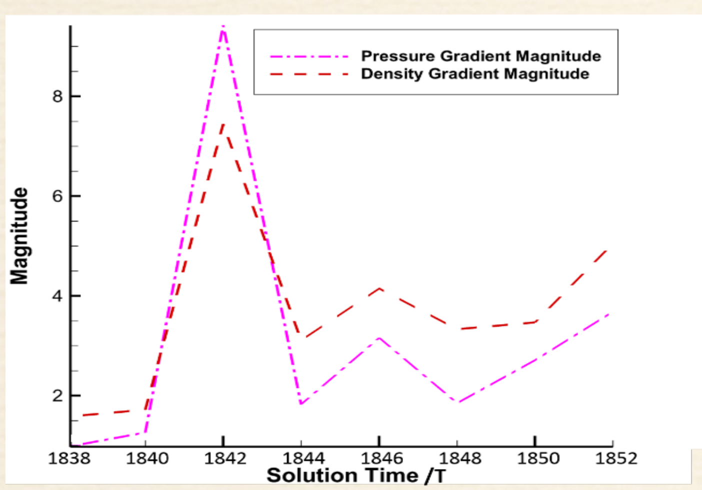


Fig. 20. $||\nabla p||$ **and** $||\nabla \rho||$ **of change at the center of ring head with solution time**

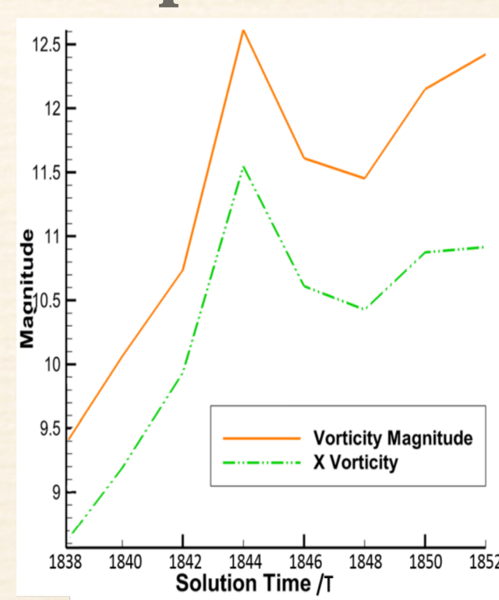
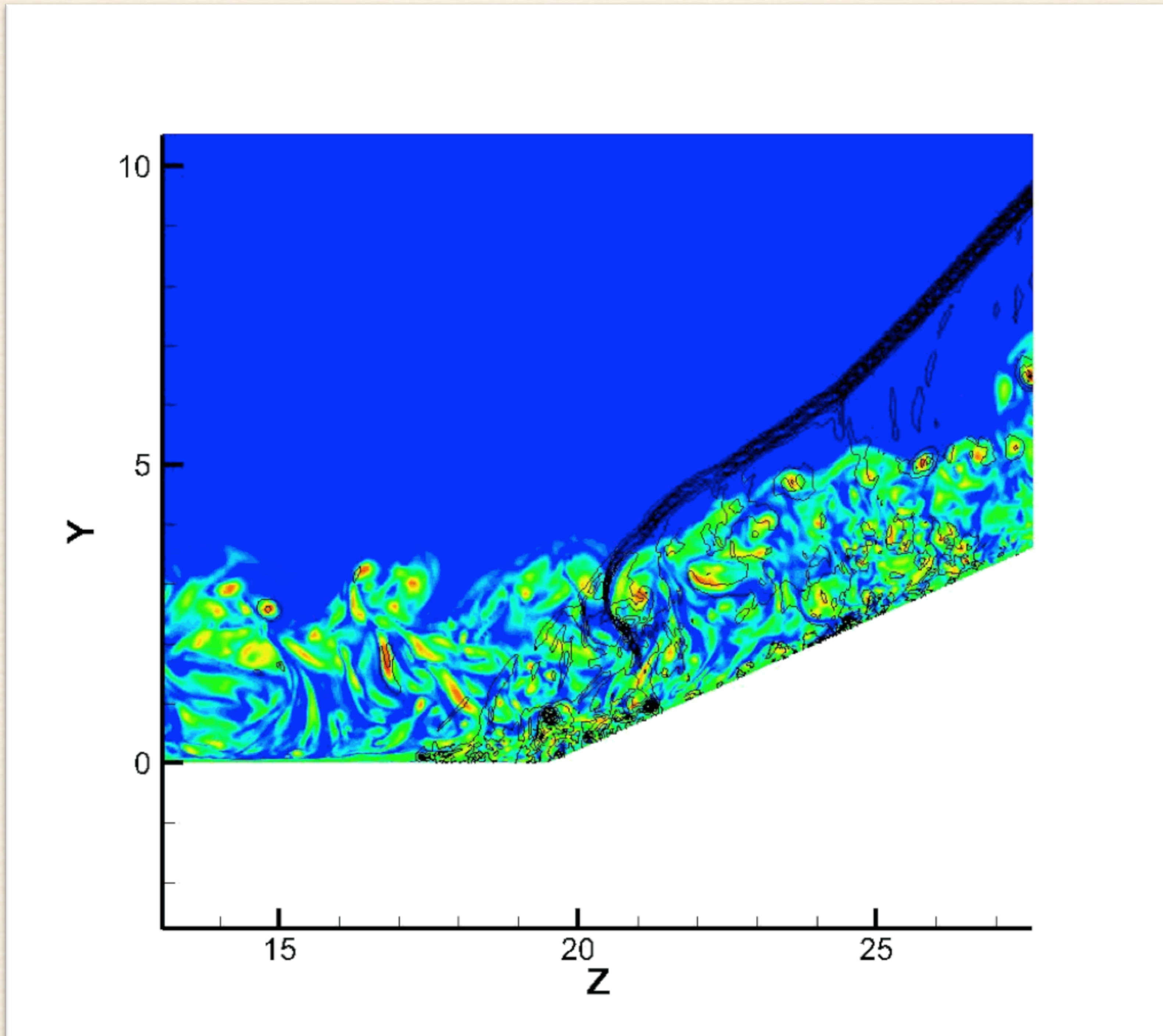


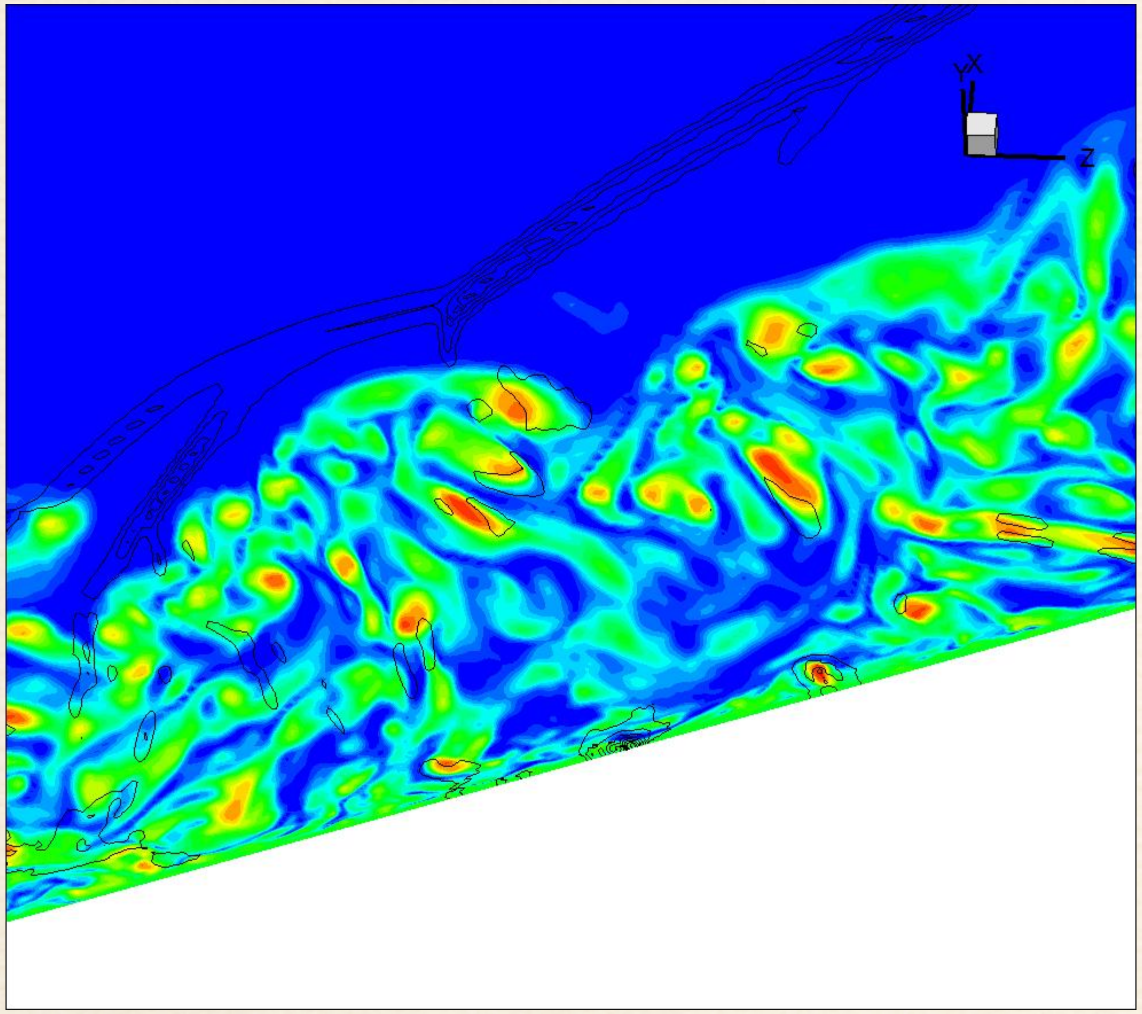
Fig. 21. Vorticity magnitude and X vorticity of change in the center of ring head with solution time

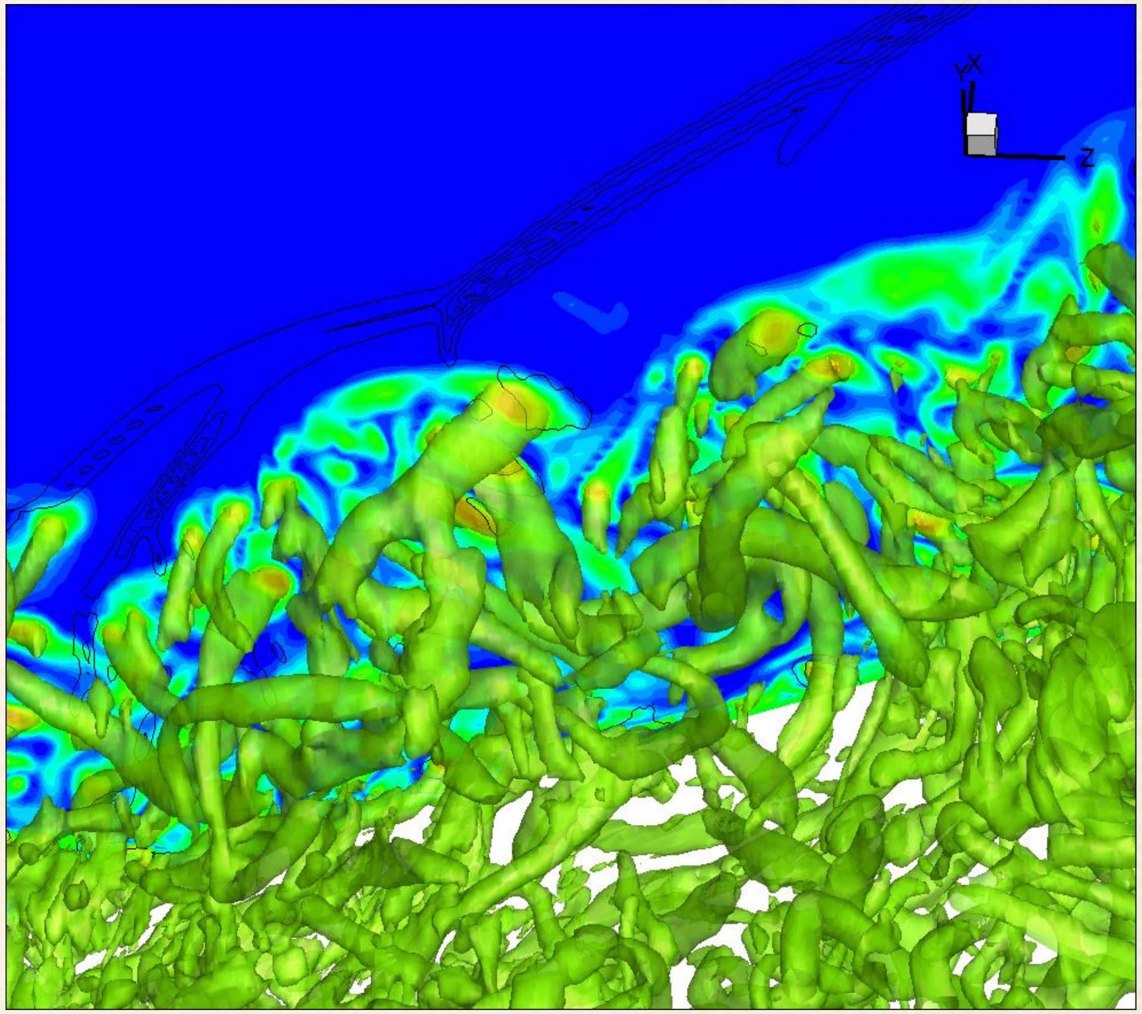
Vortex destroys shock

Ring passing through the shock on the central plane



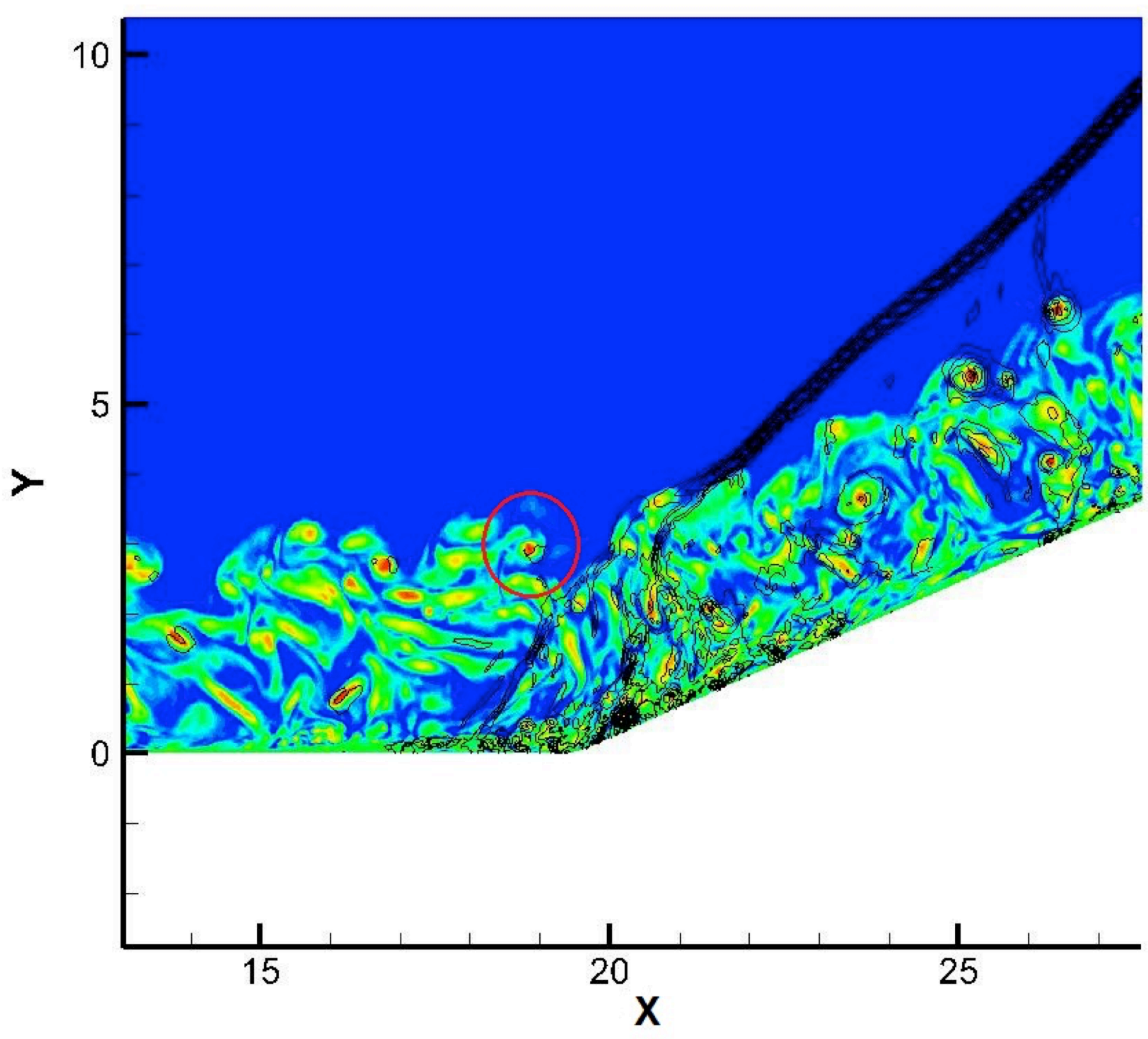
The vortex ring iso-surface



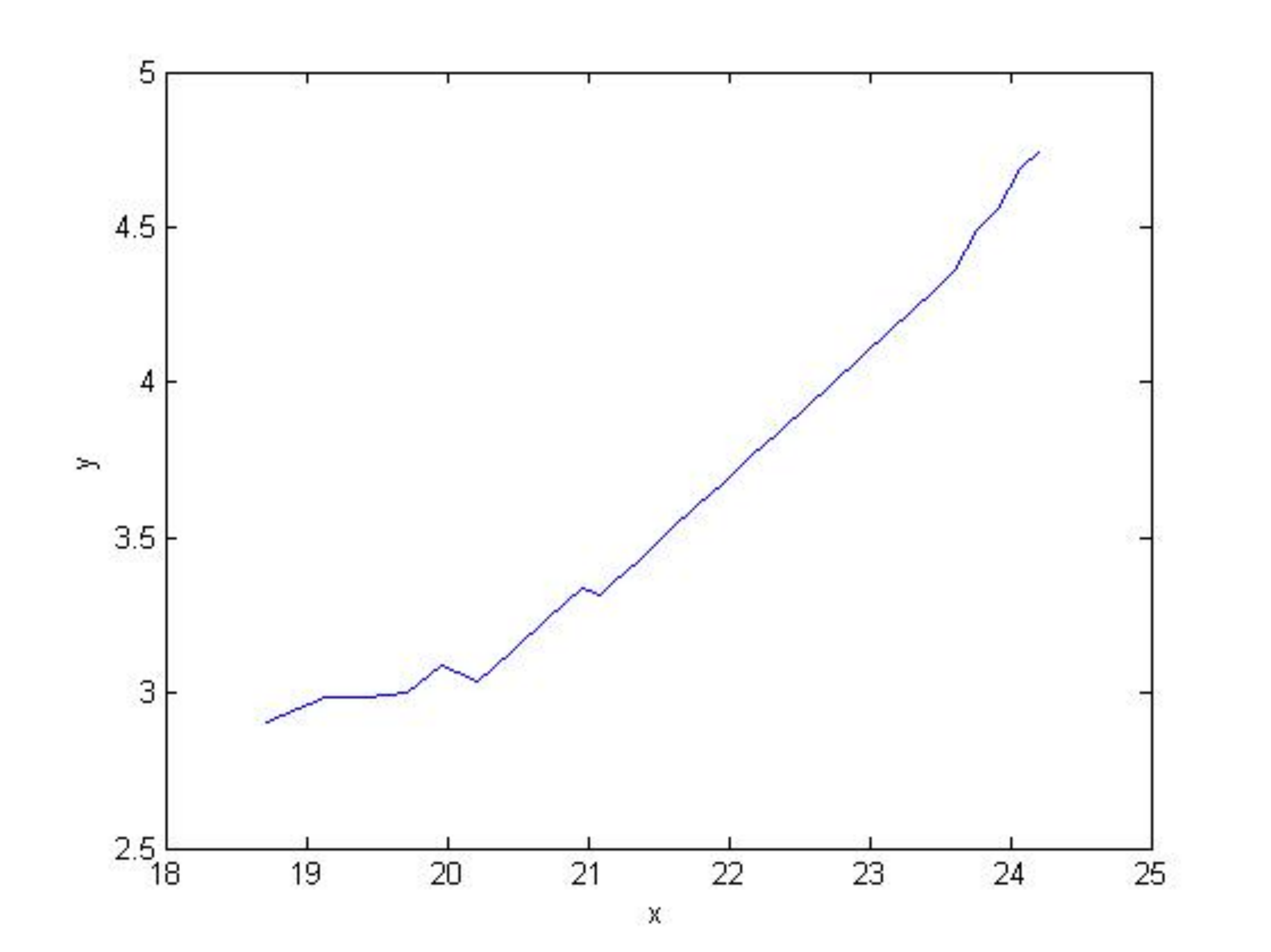


Vortex rings are captured by Ω iso-surfaces. Each red spot represents the cross section of rings with the central plane.

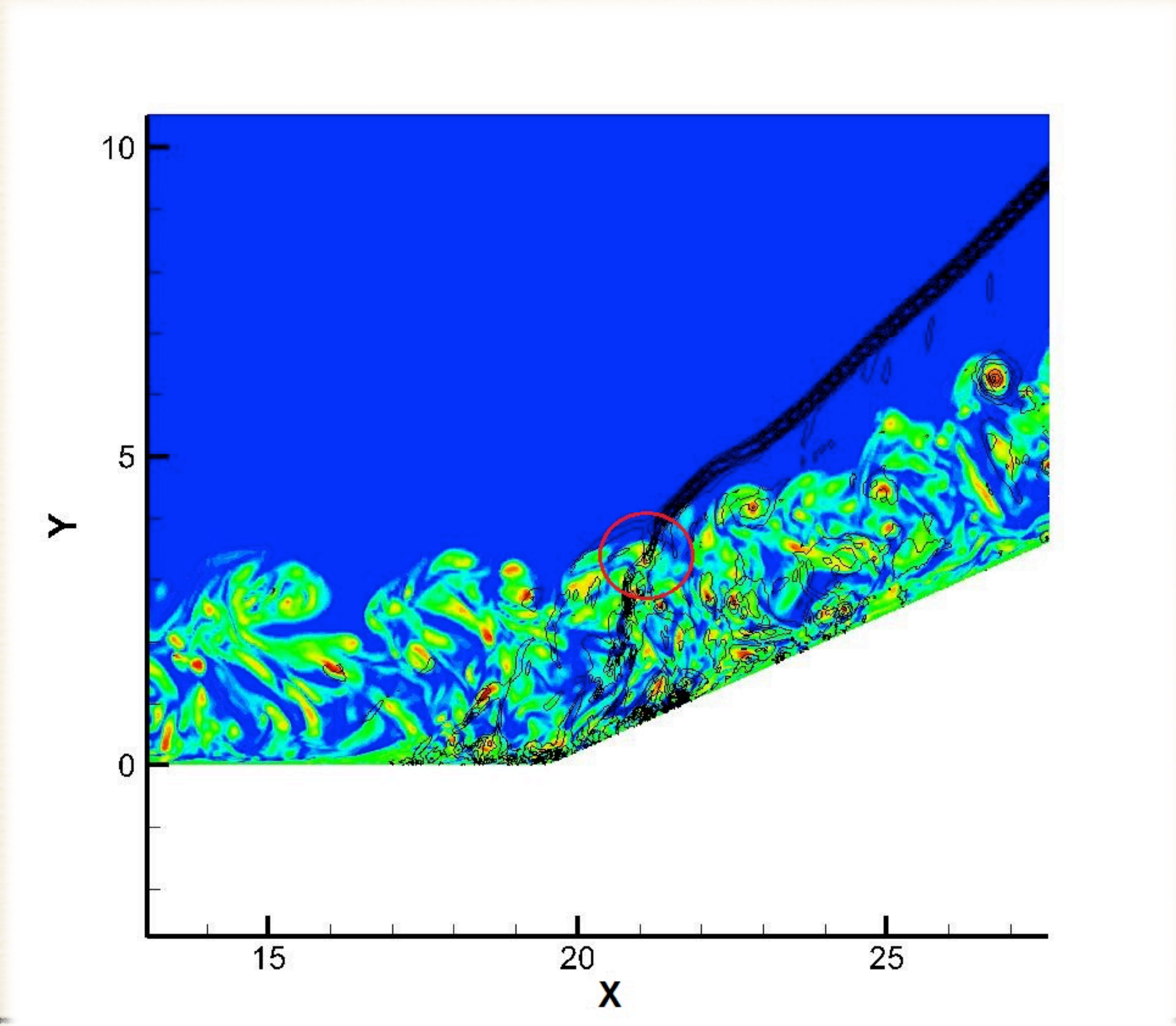
Tracking vortex ring 1 from time step 445000 to 471000



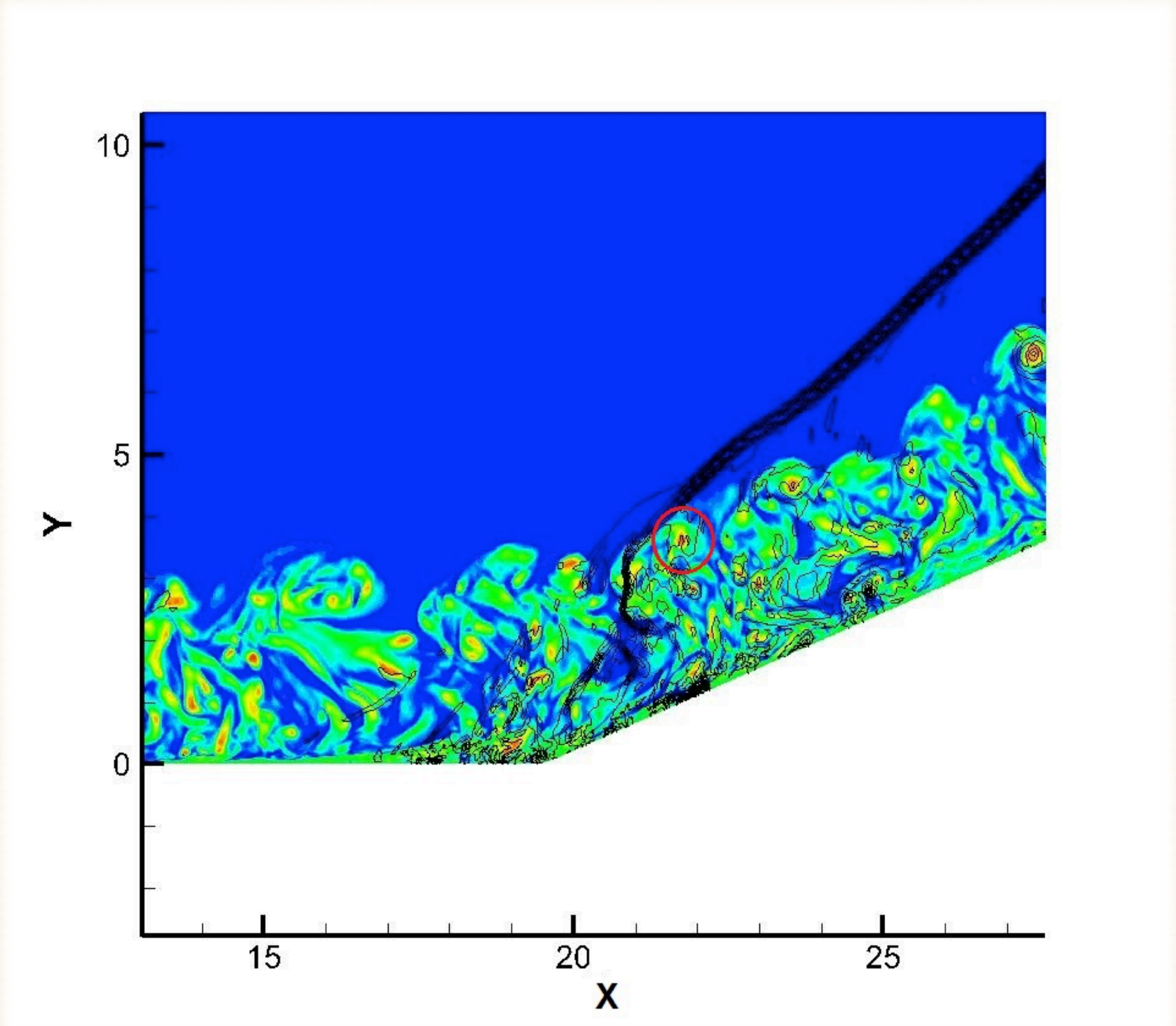
Position



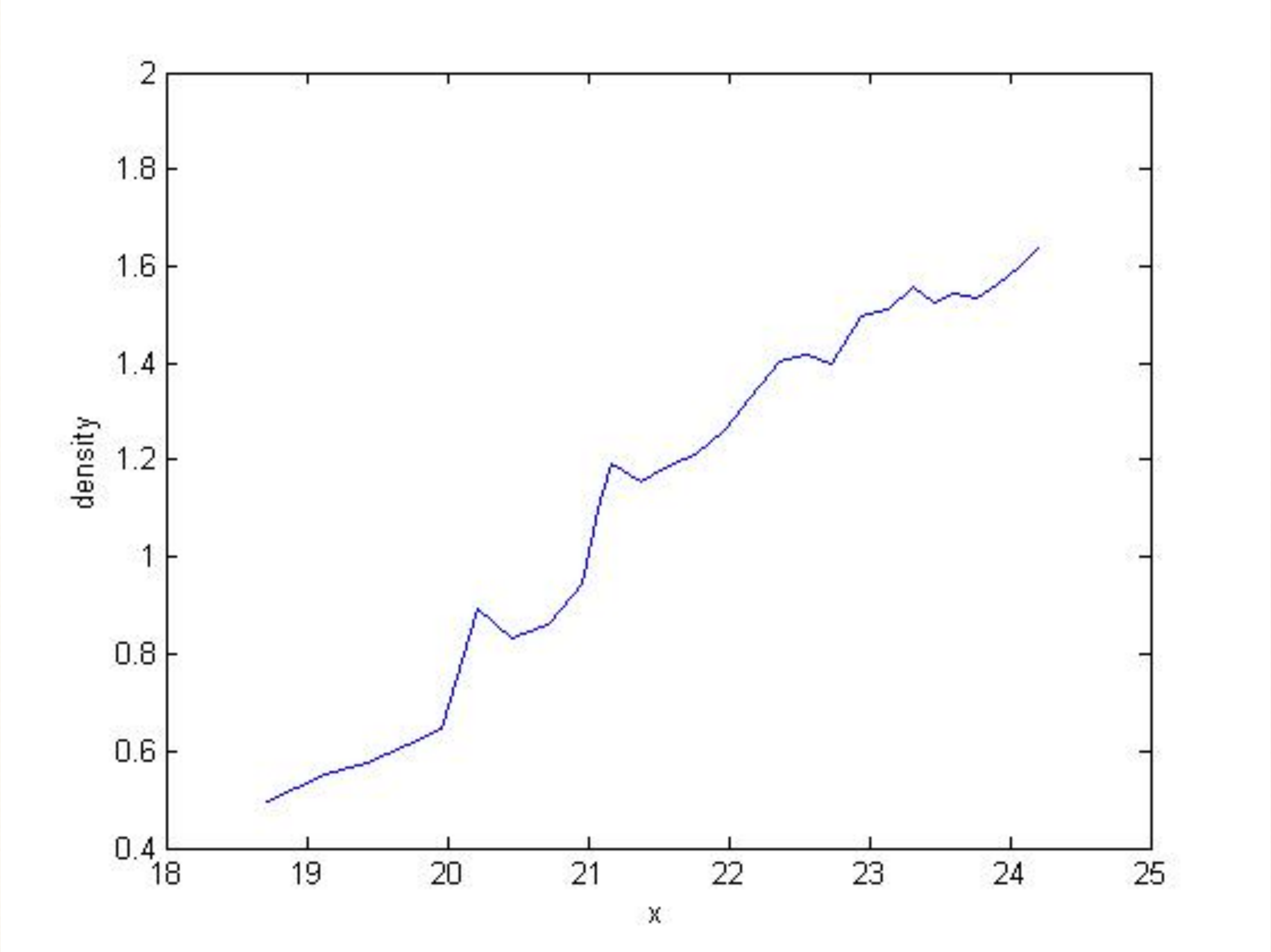
Ring hits the shock at 453500



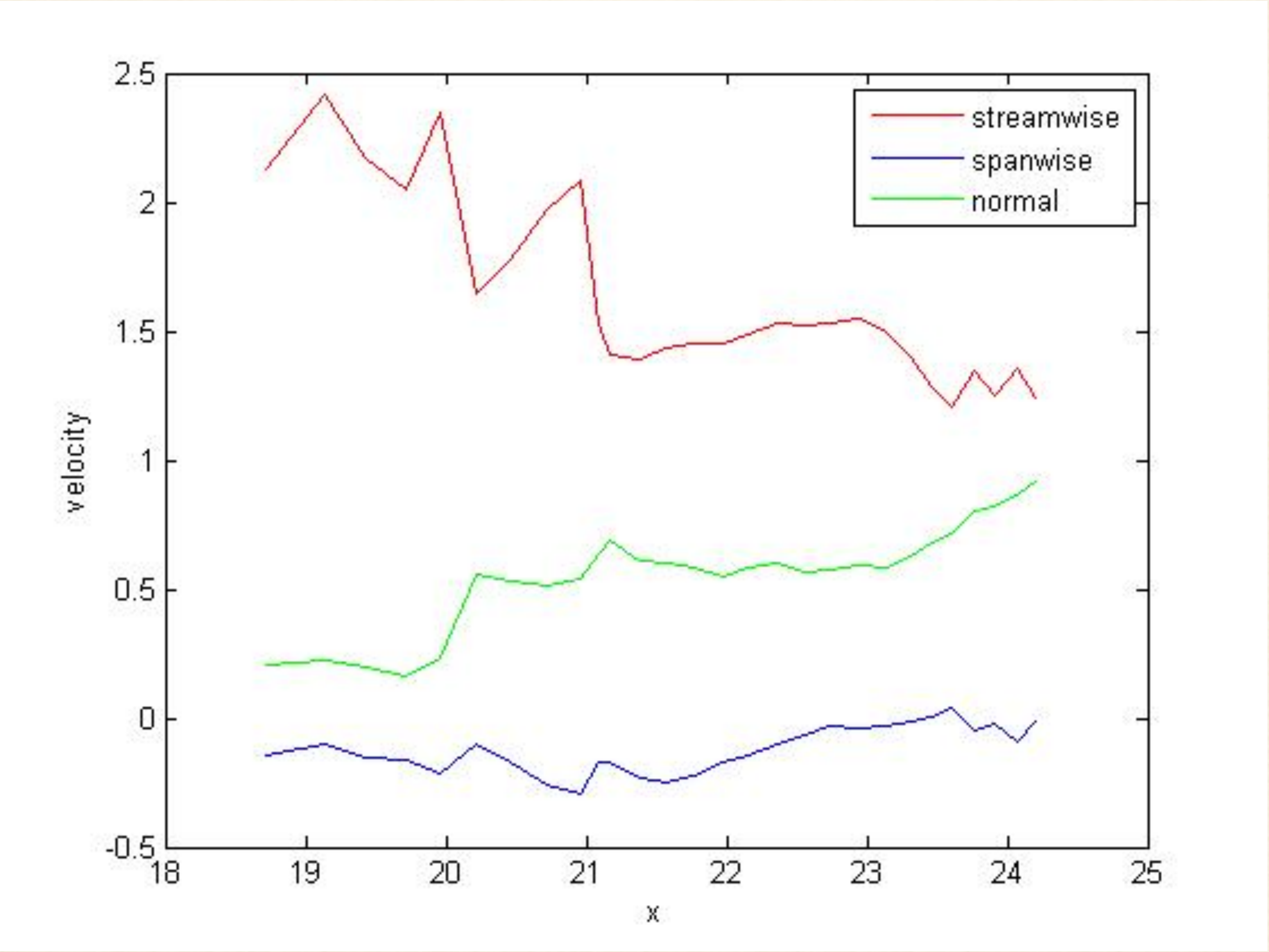
Ring passes shock at 457000



Density

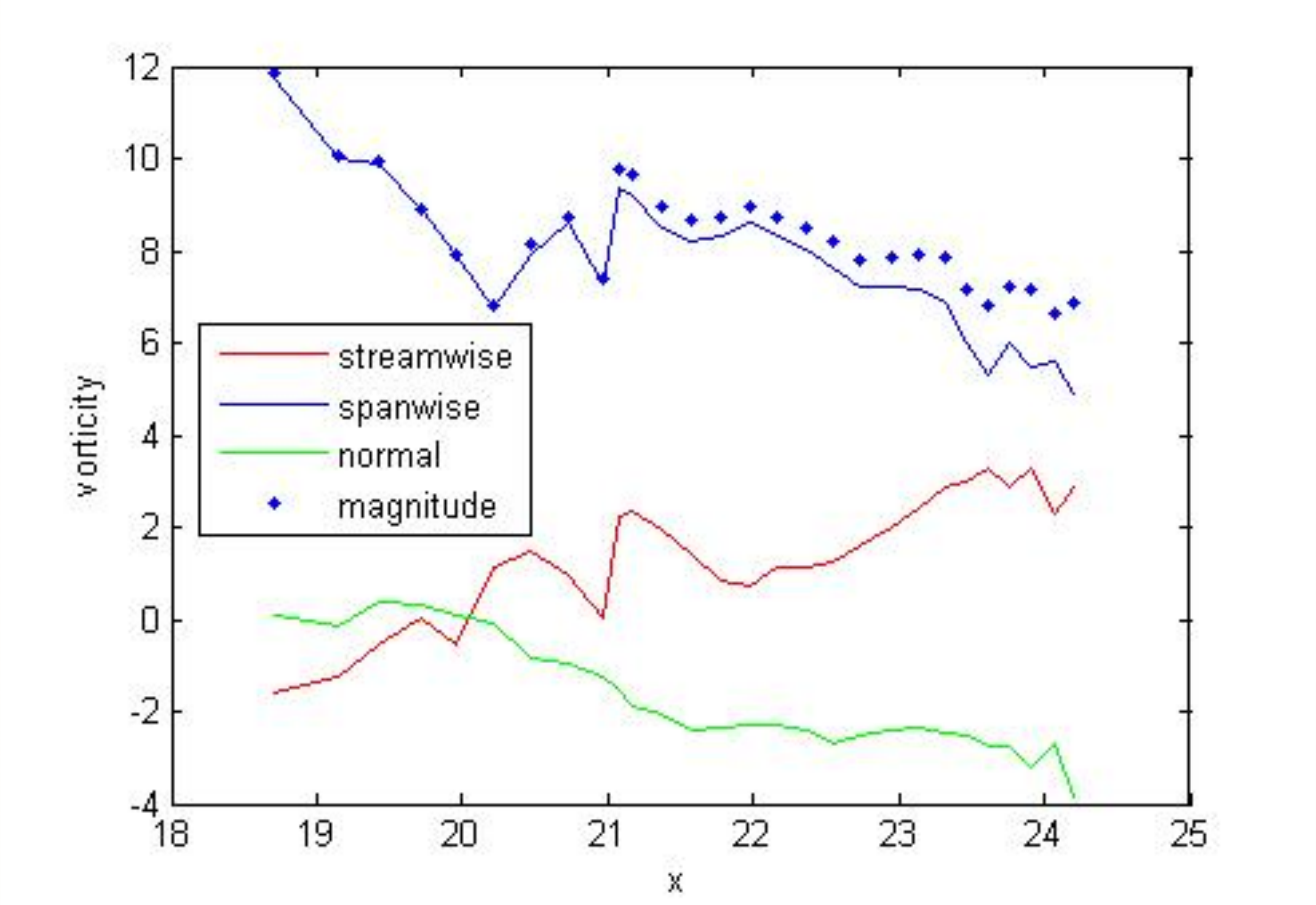


Velocity



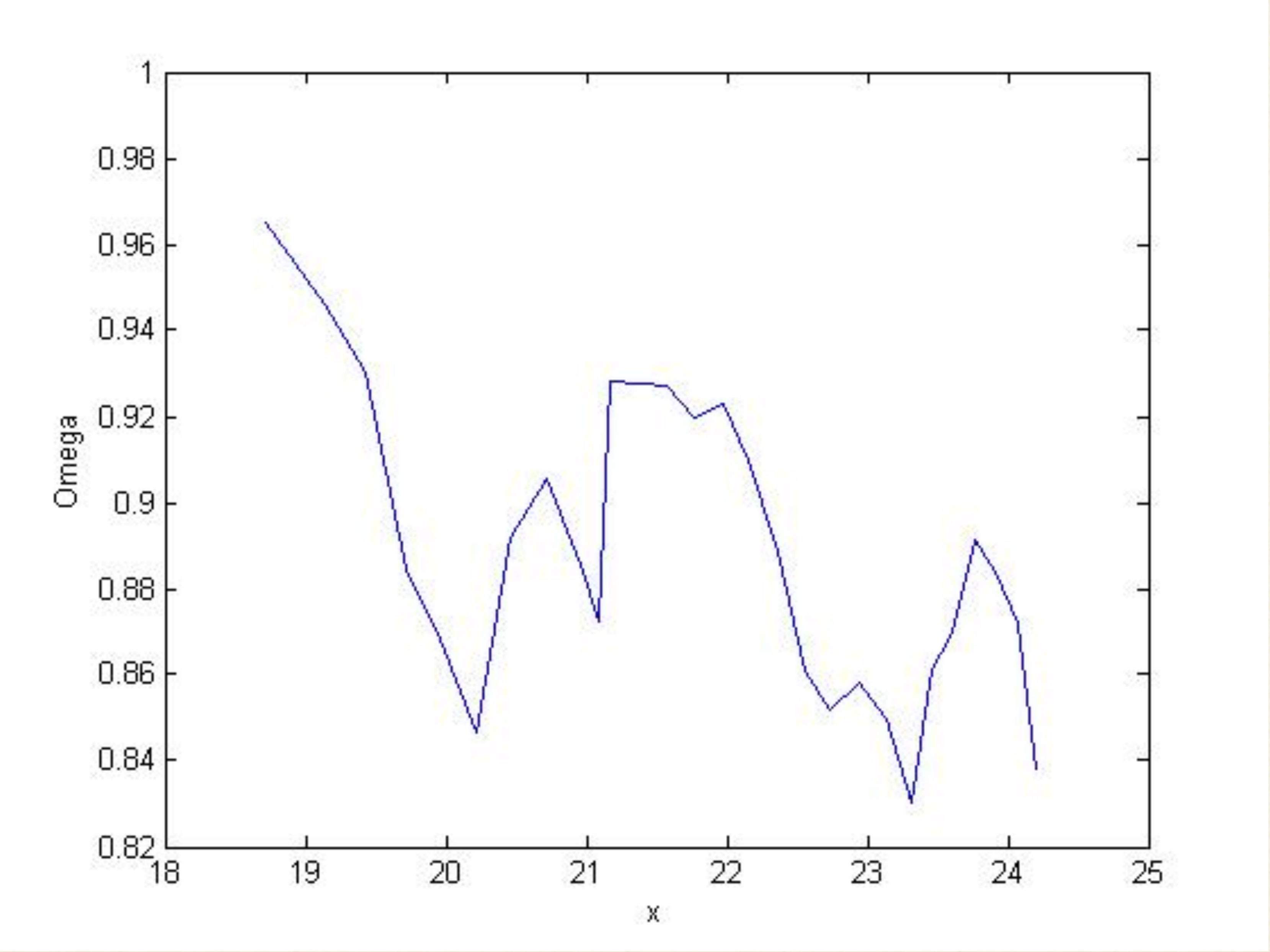
Vortex ring rolls up

Vorticity

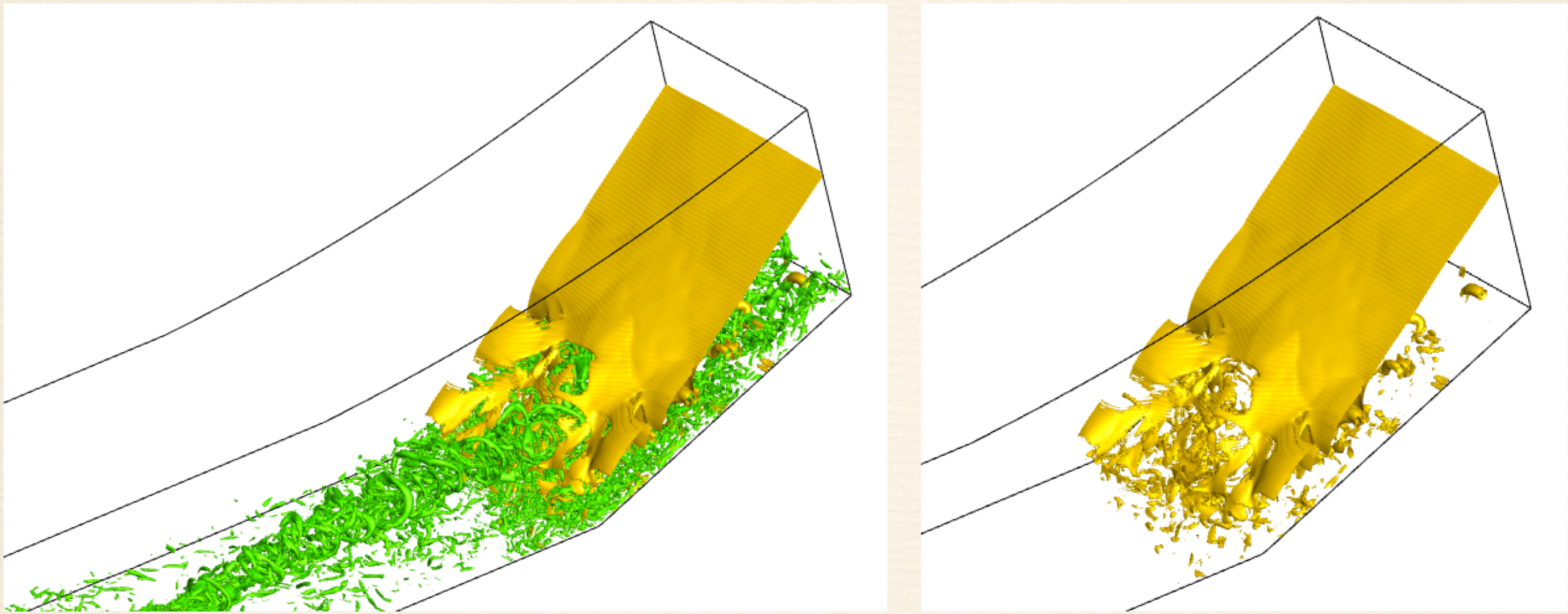


Spanwise vorticity is dominant

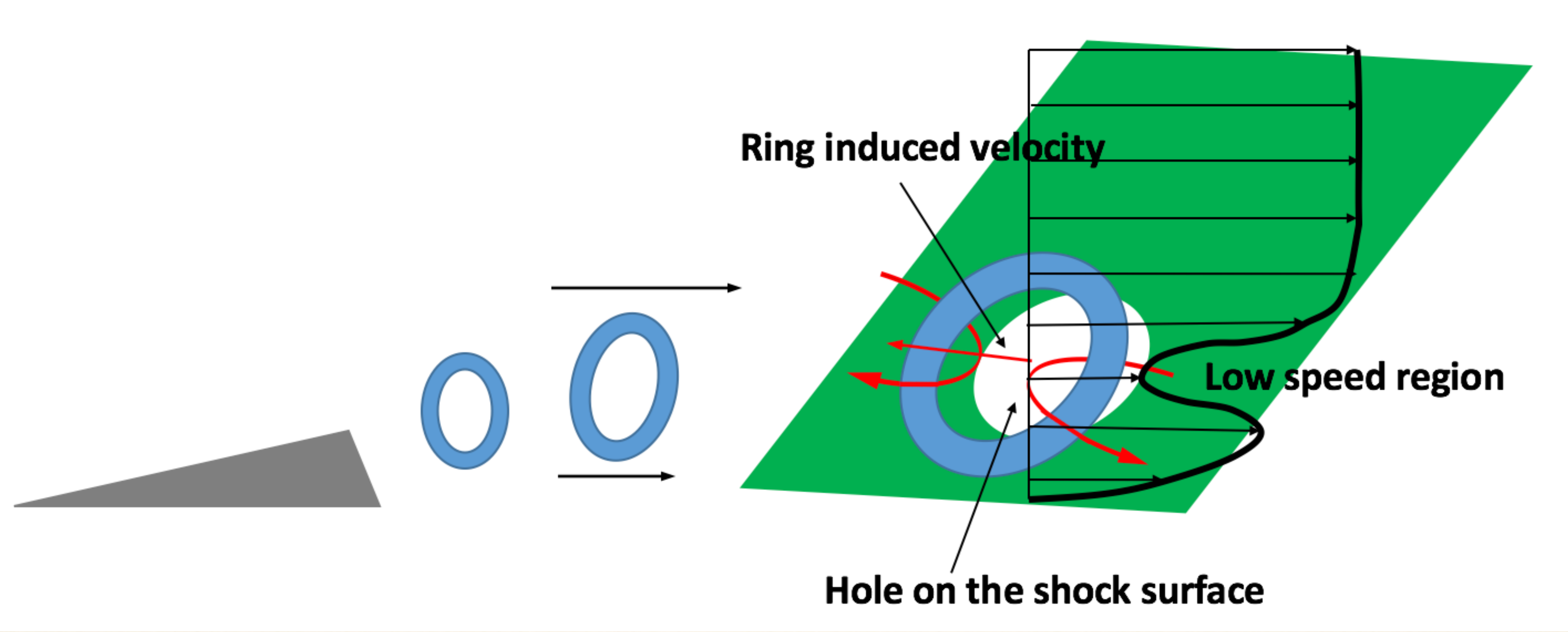
Omega (large-Rotating)



Broken Shock Surface



Mechanism Sketch

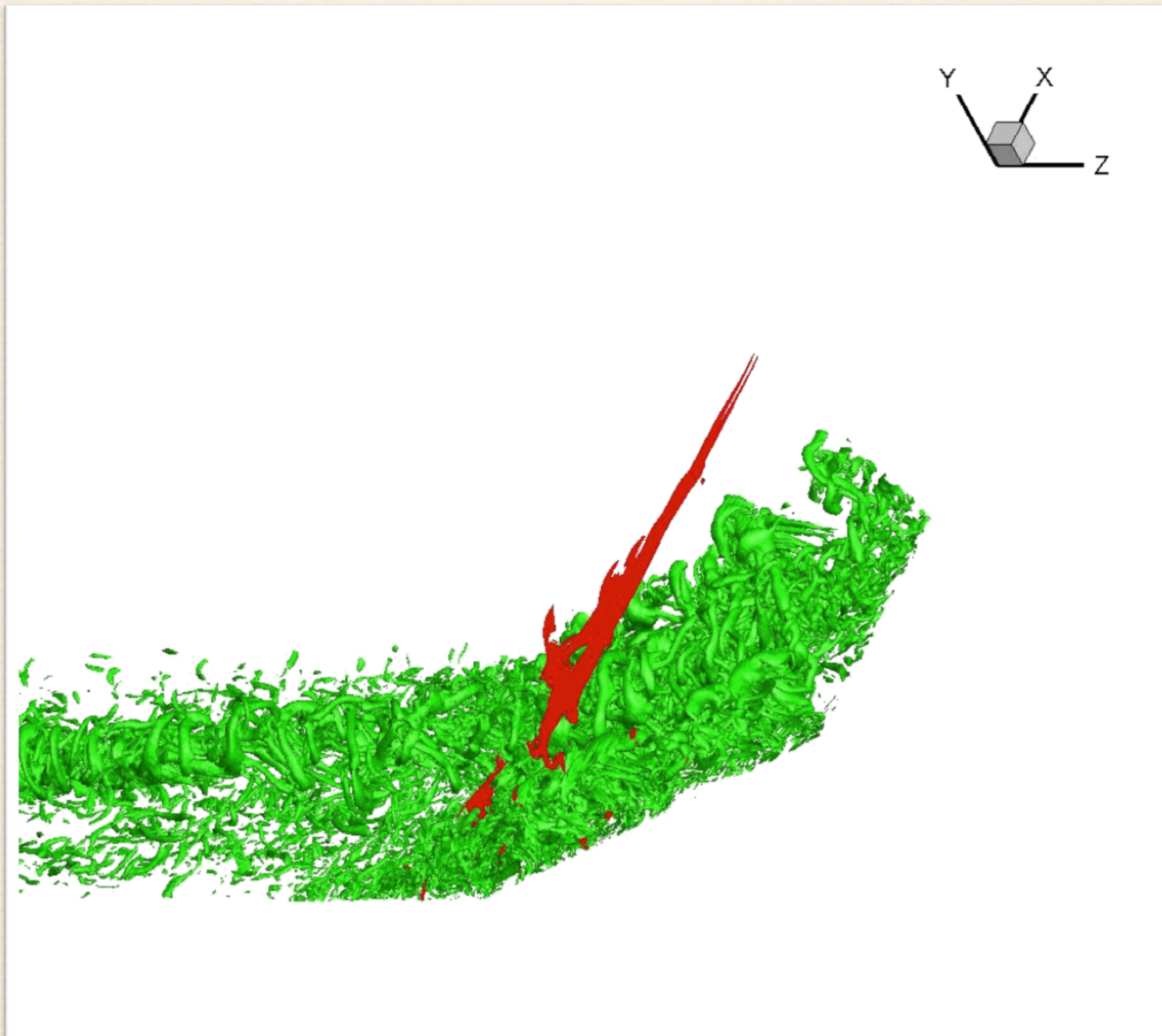


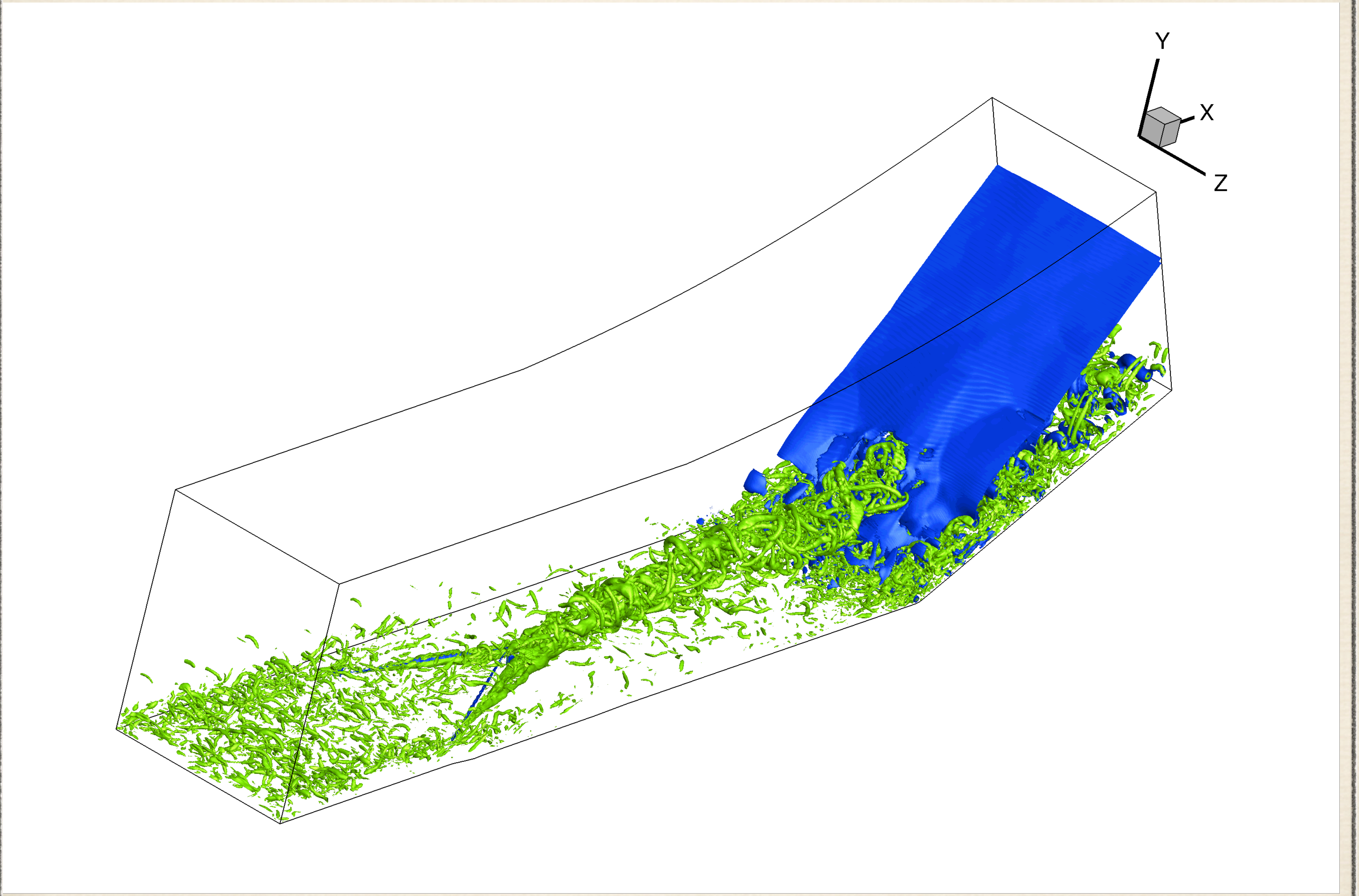
Conclusions

1. Use Omega method for vortex definition and identification
2. Shock-turbulent boundary interaction is shock-vortex interaction
3. Unsteadiness of SBLI is caused by vortices moving
4. Frequency of SBLI is determined by vortex shape and moving speed
5. The MVG for SBLI is vortex ring generation, which can destroy the shock
6. MVG can generate over 100,000 vortices/per second
7. Each ring can rotate over 64,000 circles/per second like solid body (Omega=0.95)
8. SBLI control – vortex ring generation control and momentum deficit control

Thank you

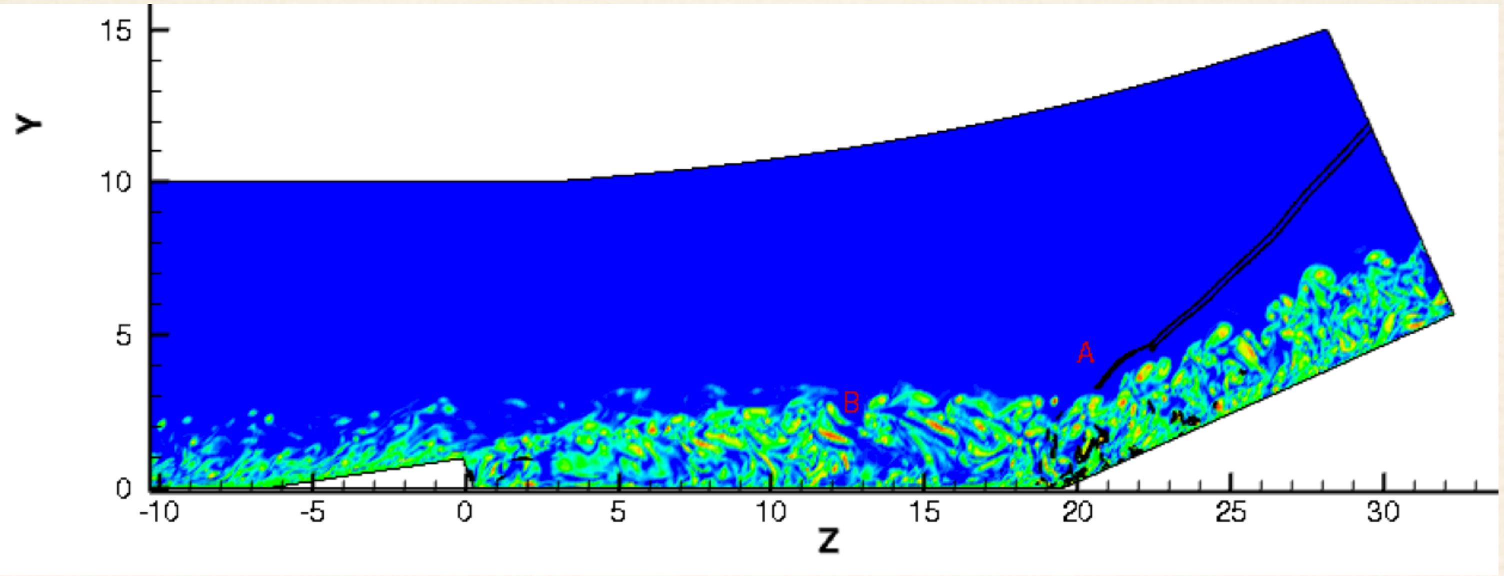
Study on evolution of vortex rings passing through shock wave

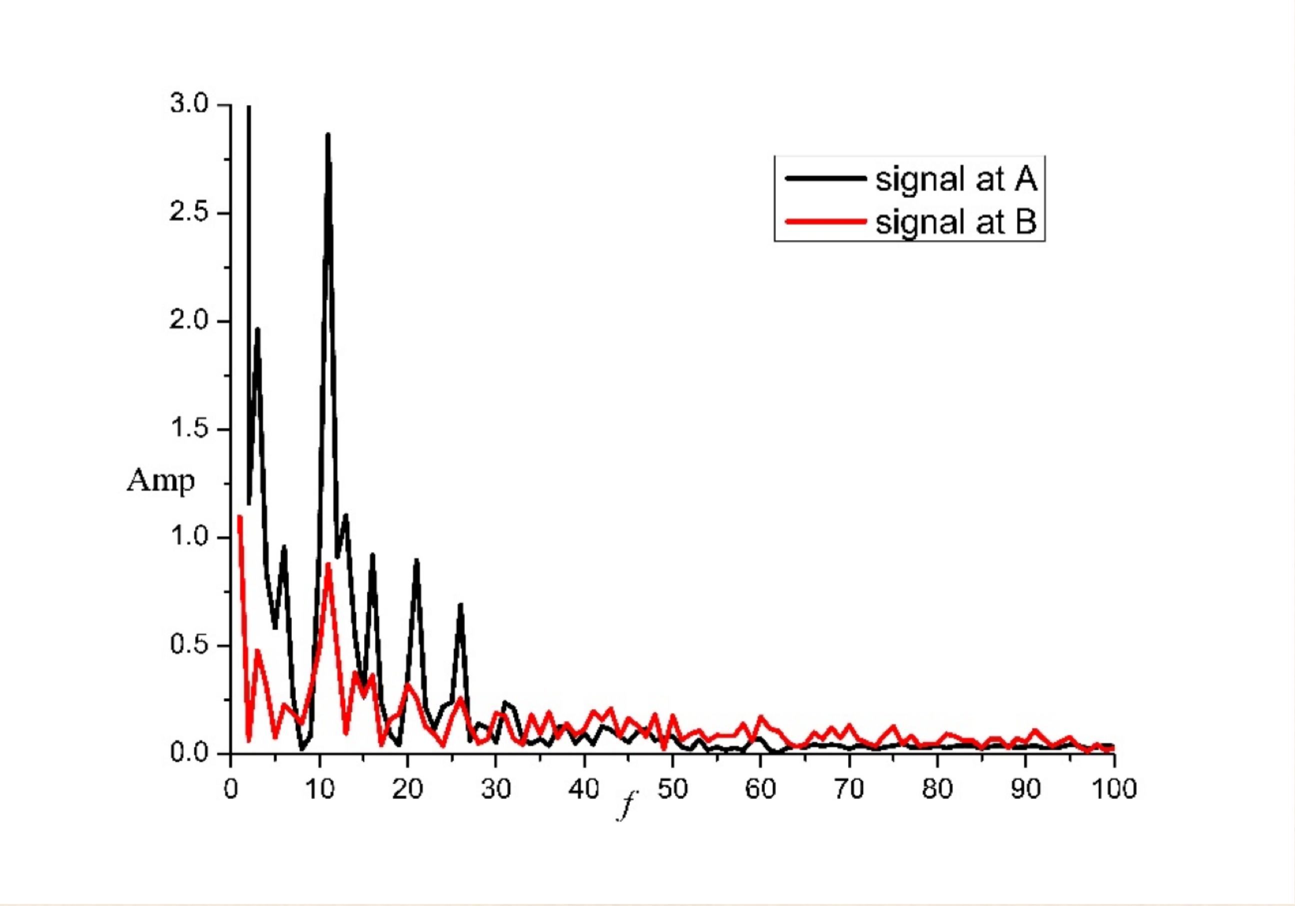




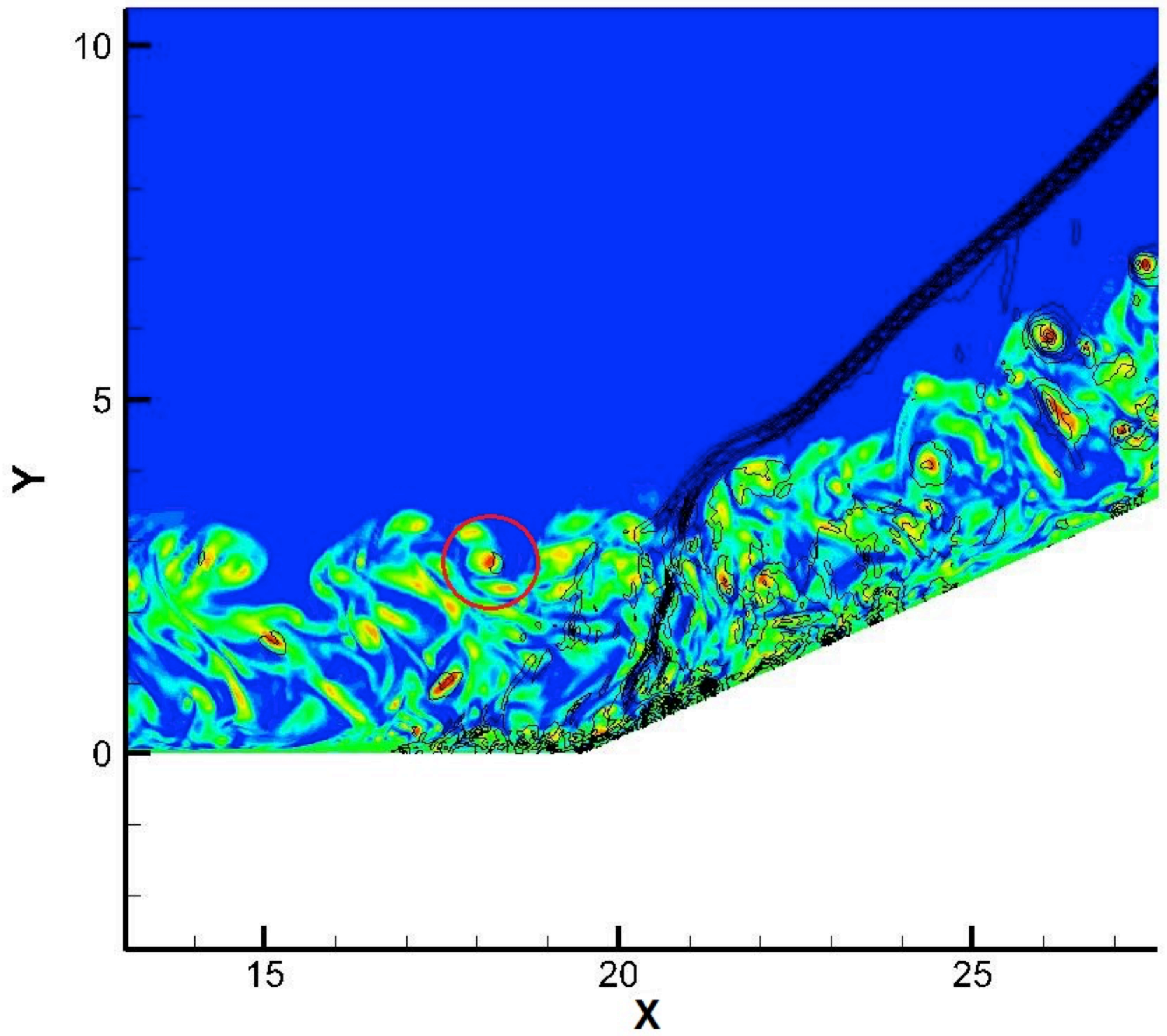
Y
X
Z

Frequency Analysis

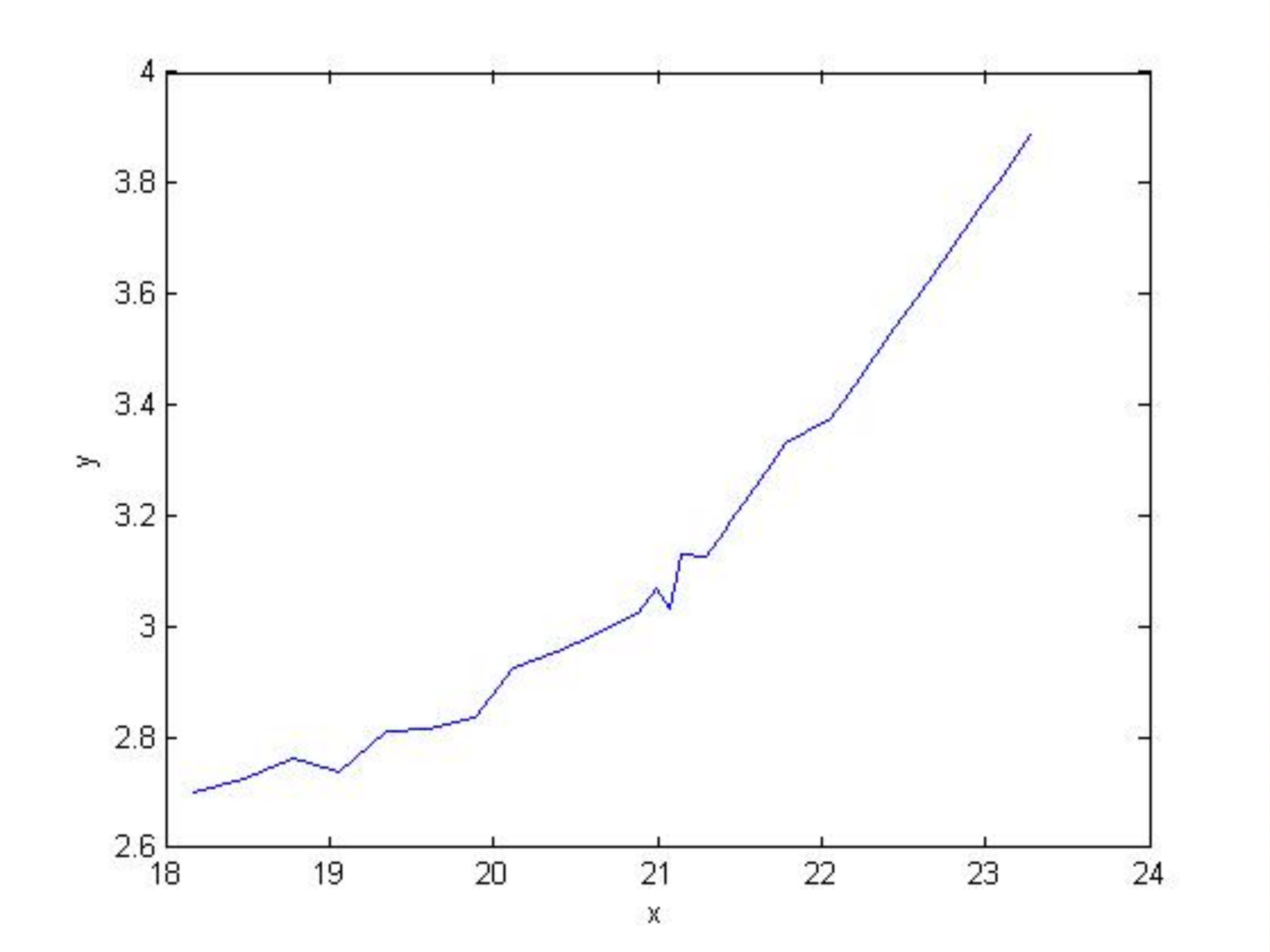




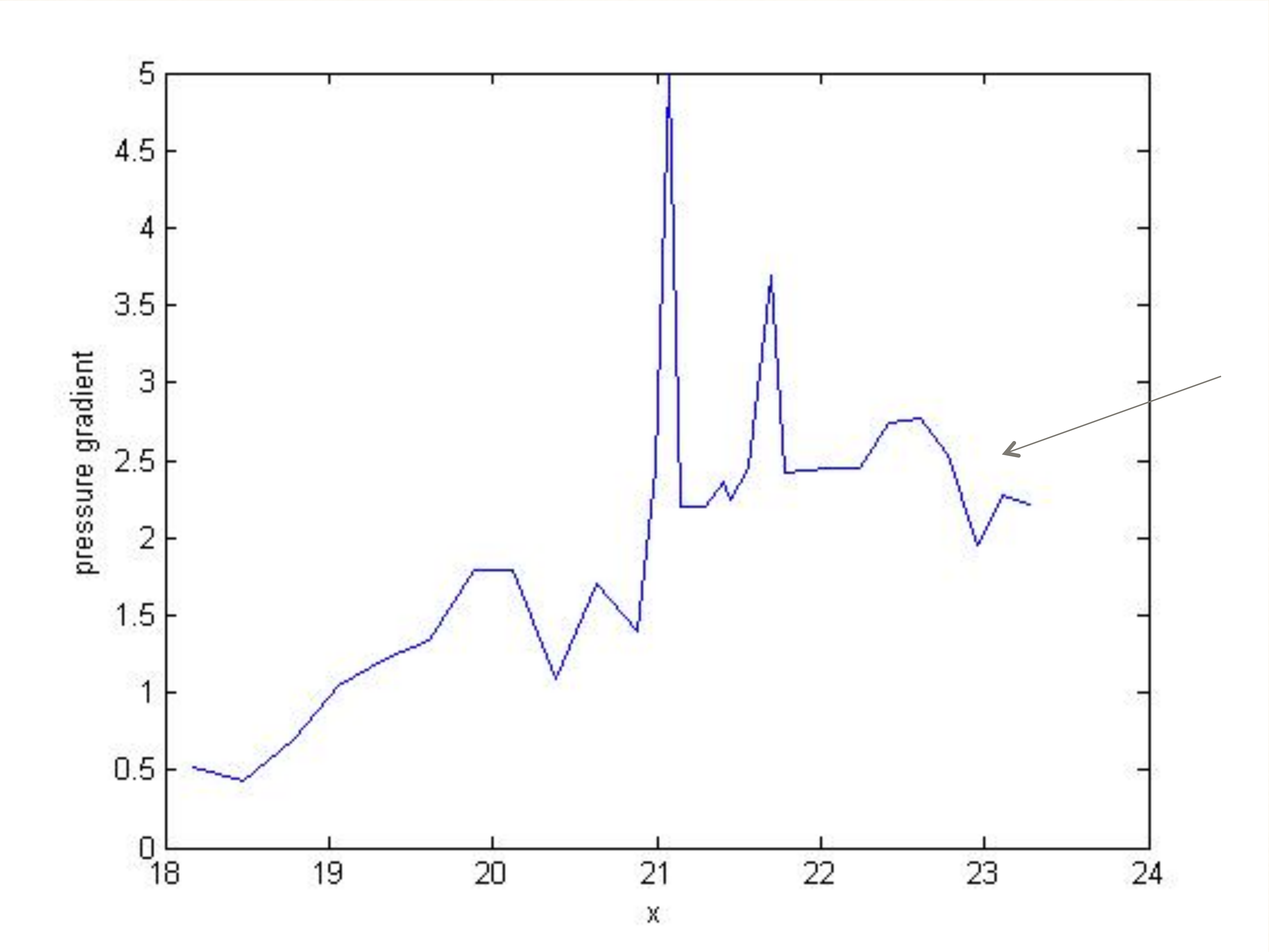
Tracking vortex ring 2 from 450000 to 473000 with shock break



Position

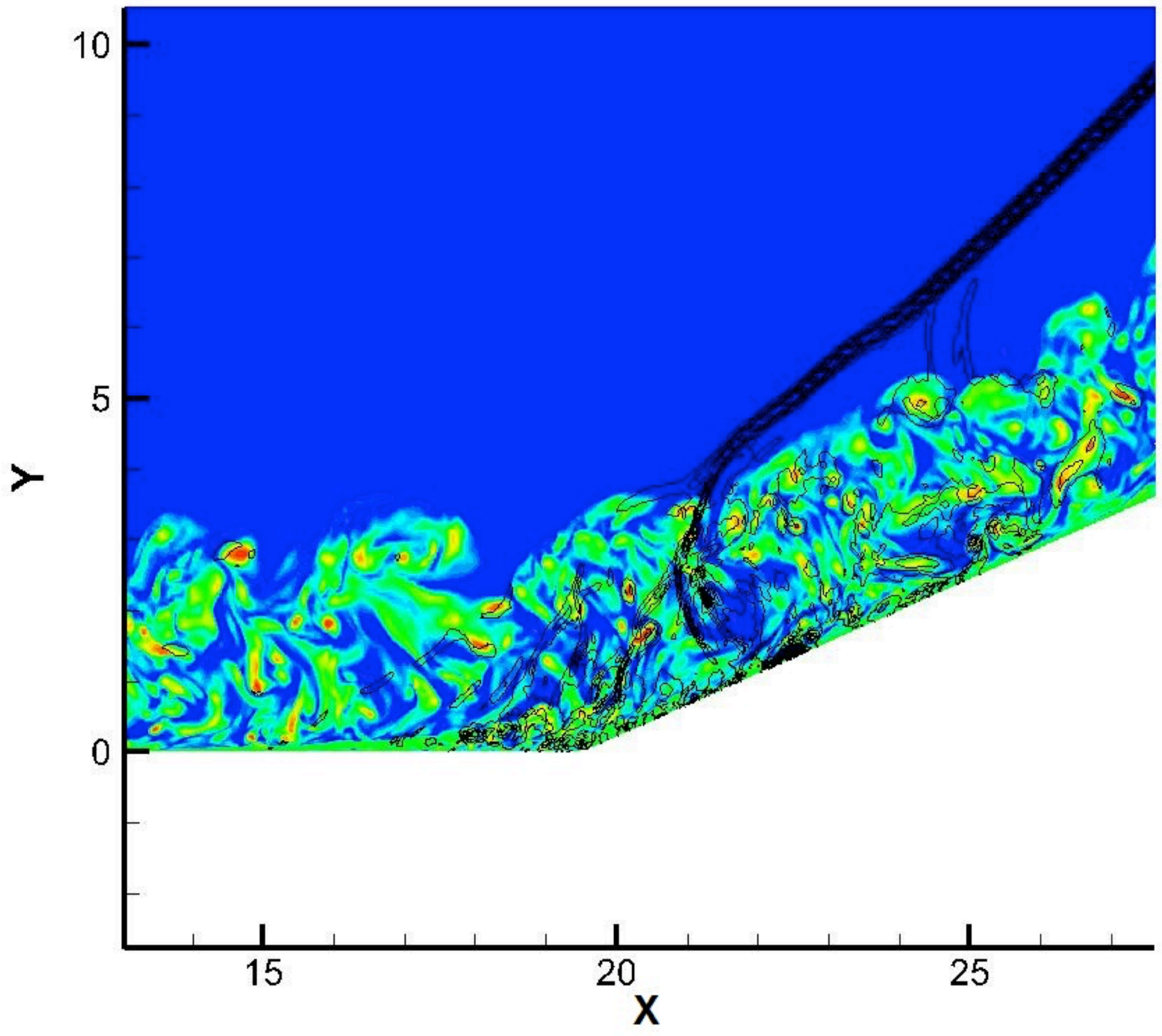


Shock

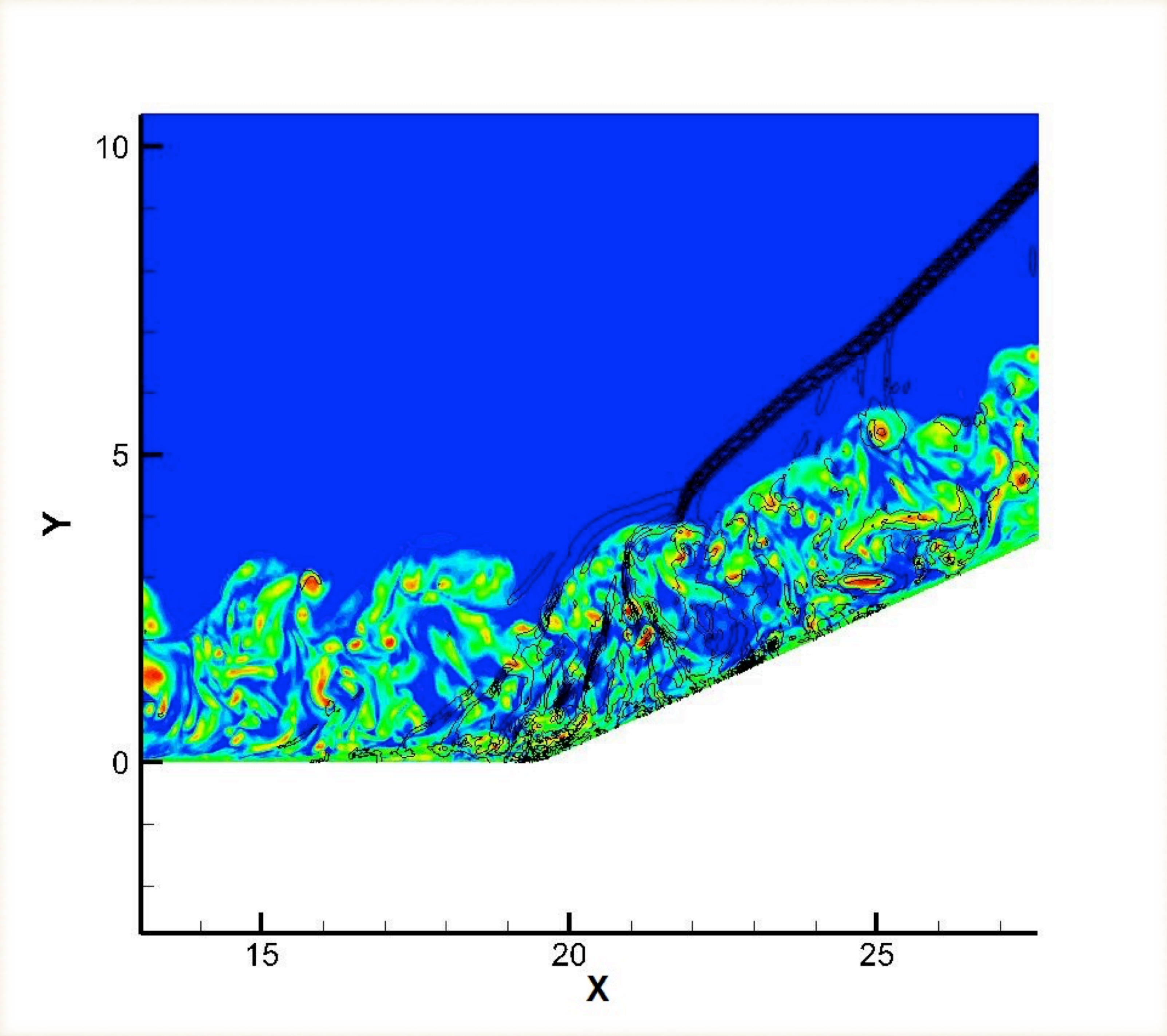


Ring surround by weak shock

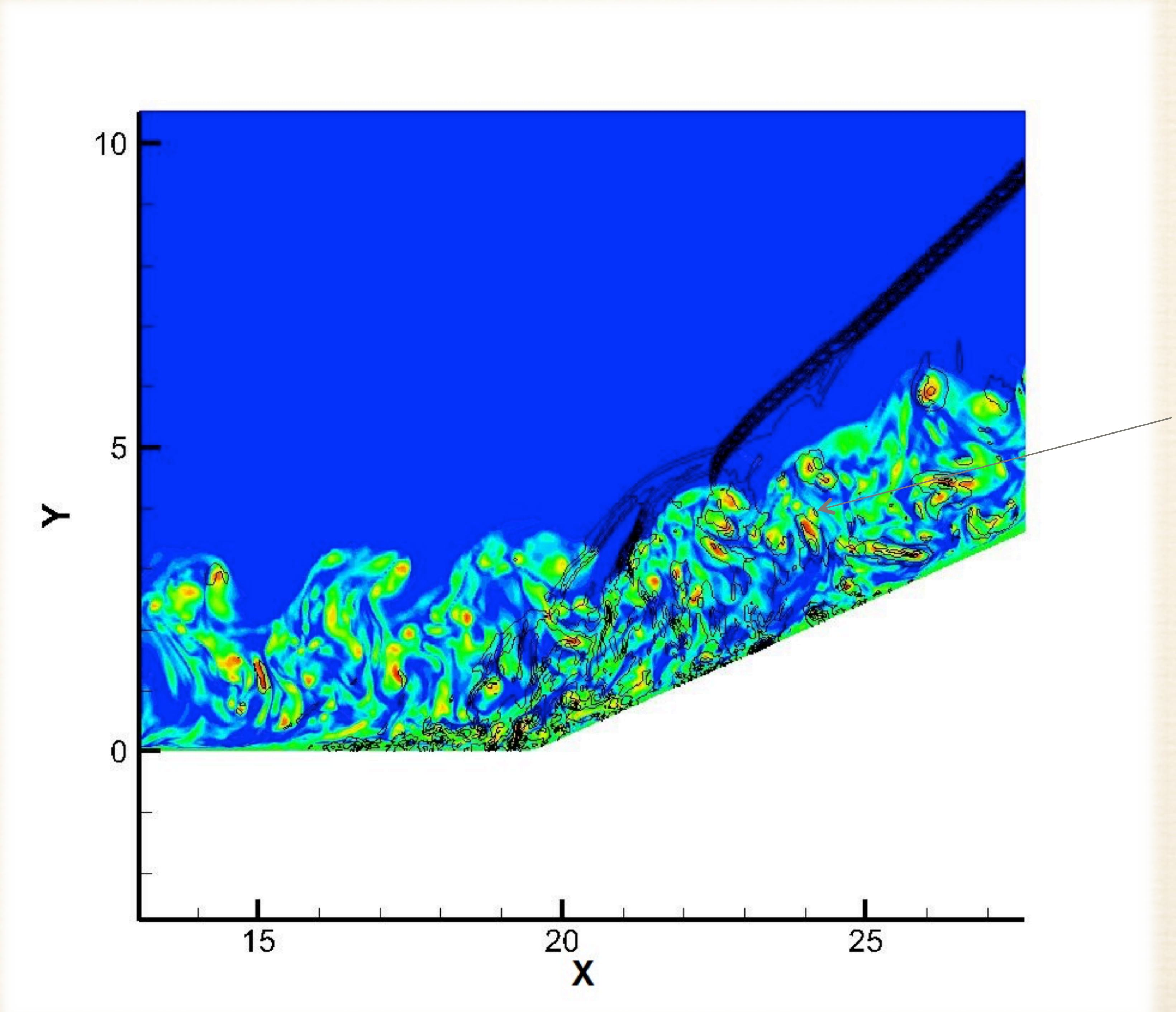
Ring hits the shock at 461000



Shock distorting at 465000

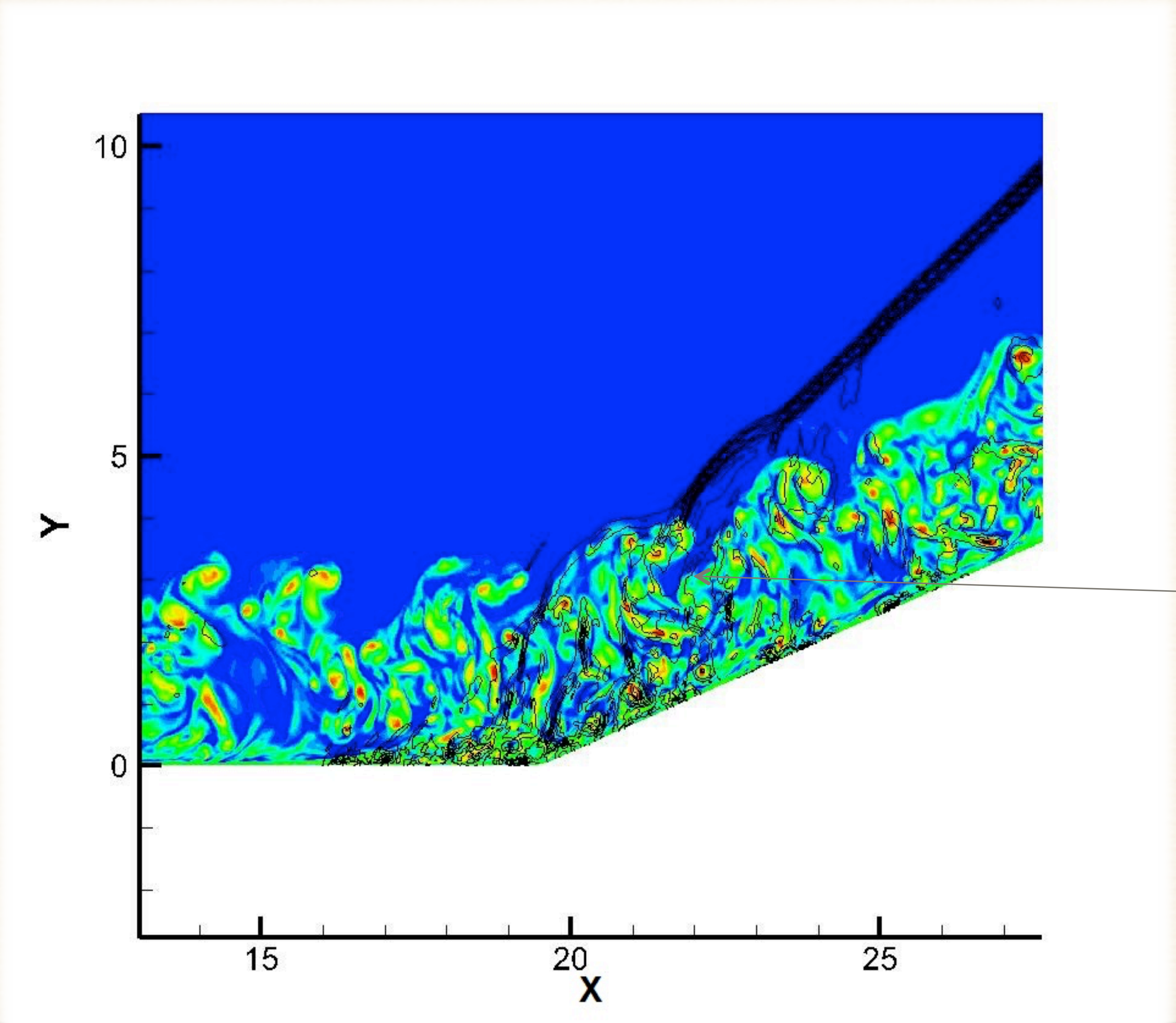


Shock breaking at 470000



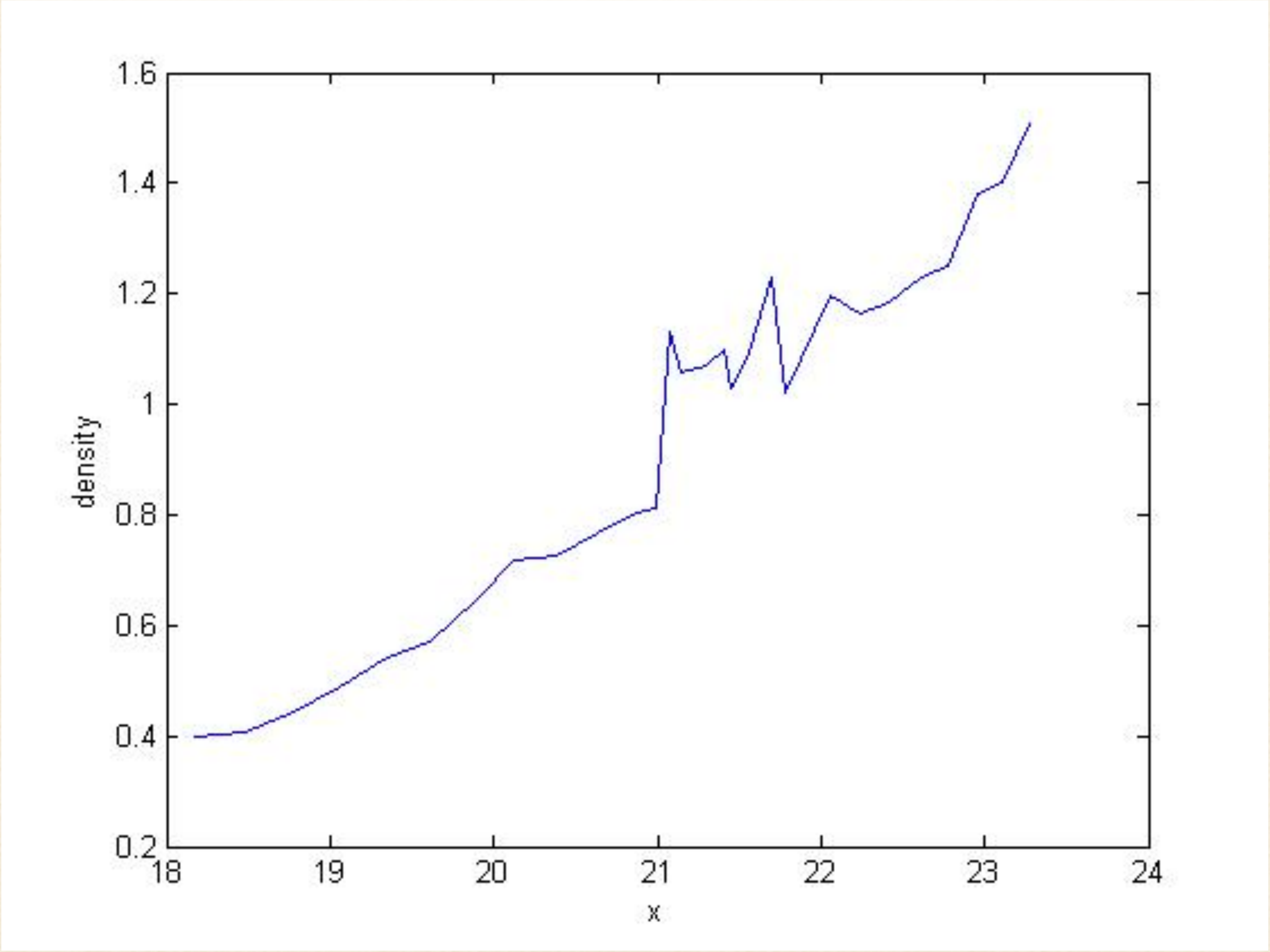
Weak shock

Upper shock recovery at 476000

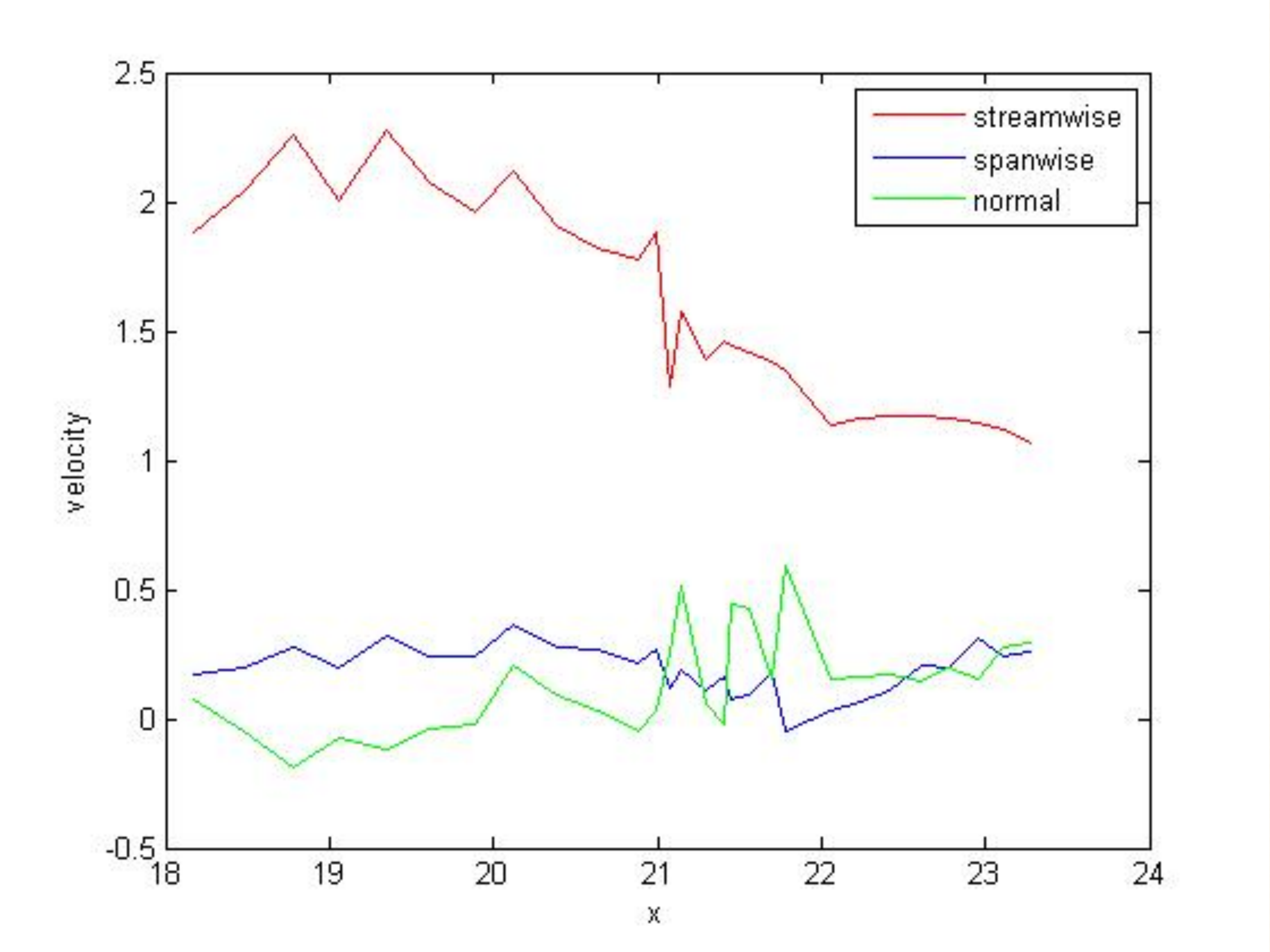


The lower shock gets distortion when interacting with vortex rings.

Density

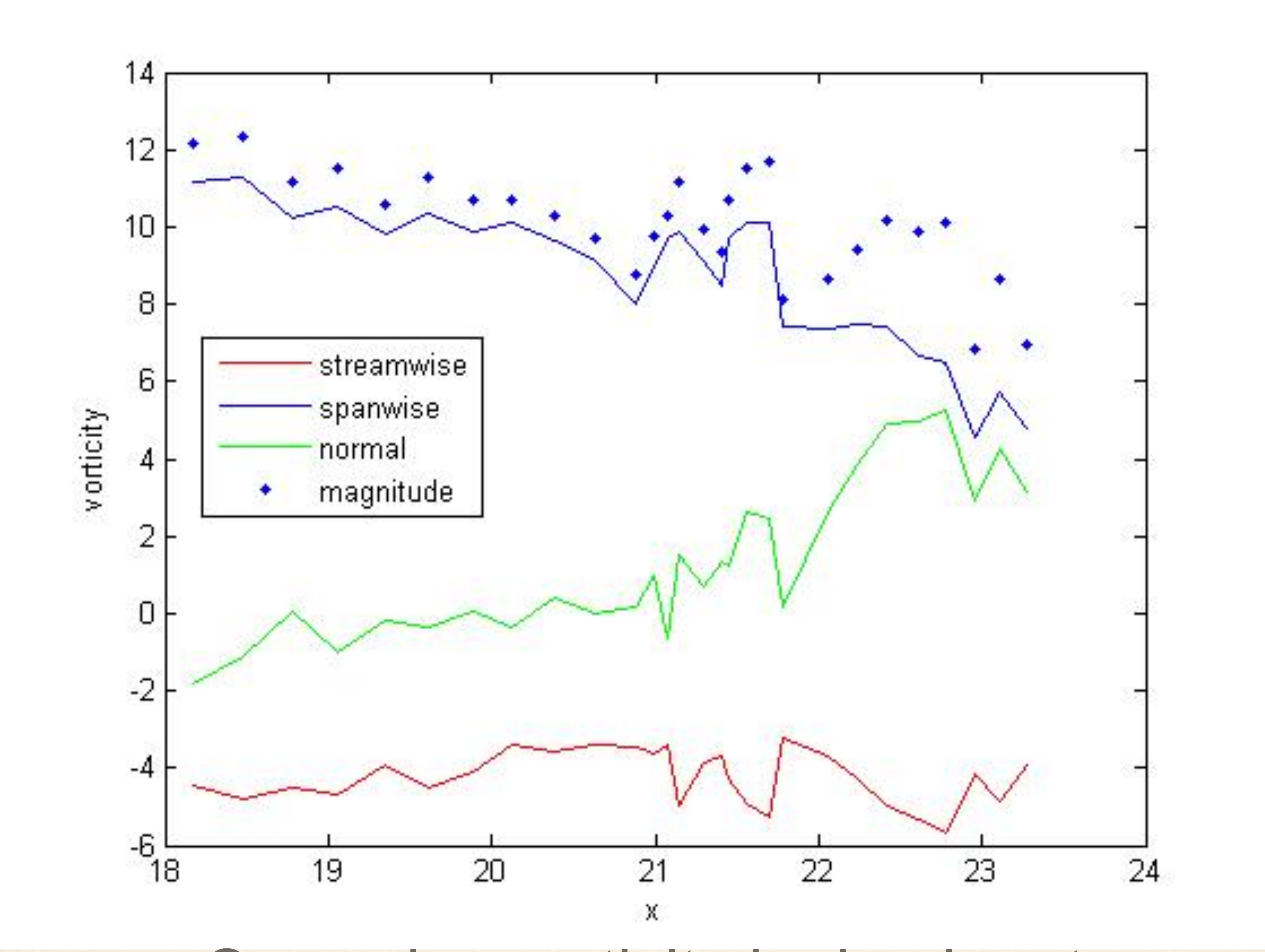


Velocity



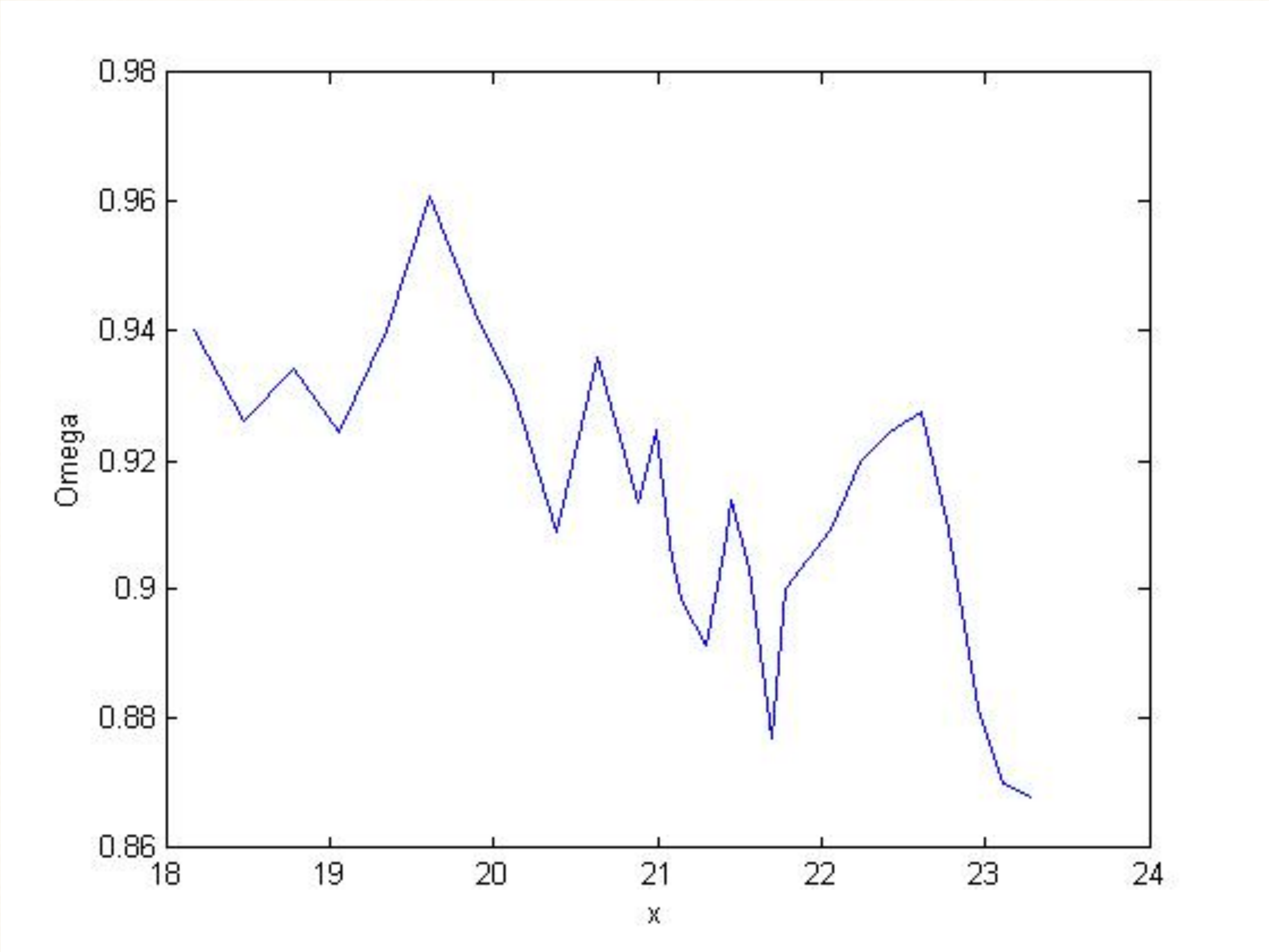
Ring hold back by the shock

Vorticity



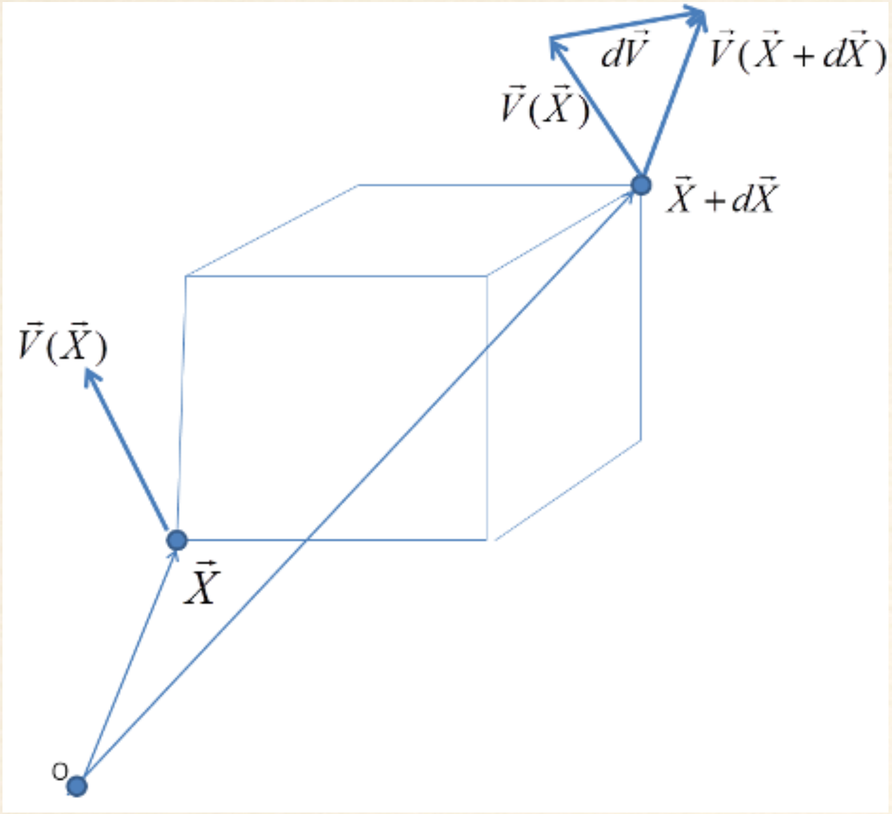
Spanwise vorticity is dominant

Omega



Thanks

Helmholtz decomposition revisit



According to Helmholtz
Fluid particle motion can be decomposed as translation, deformation and rotation

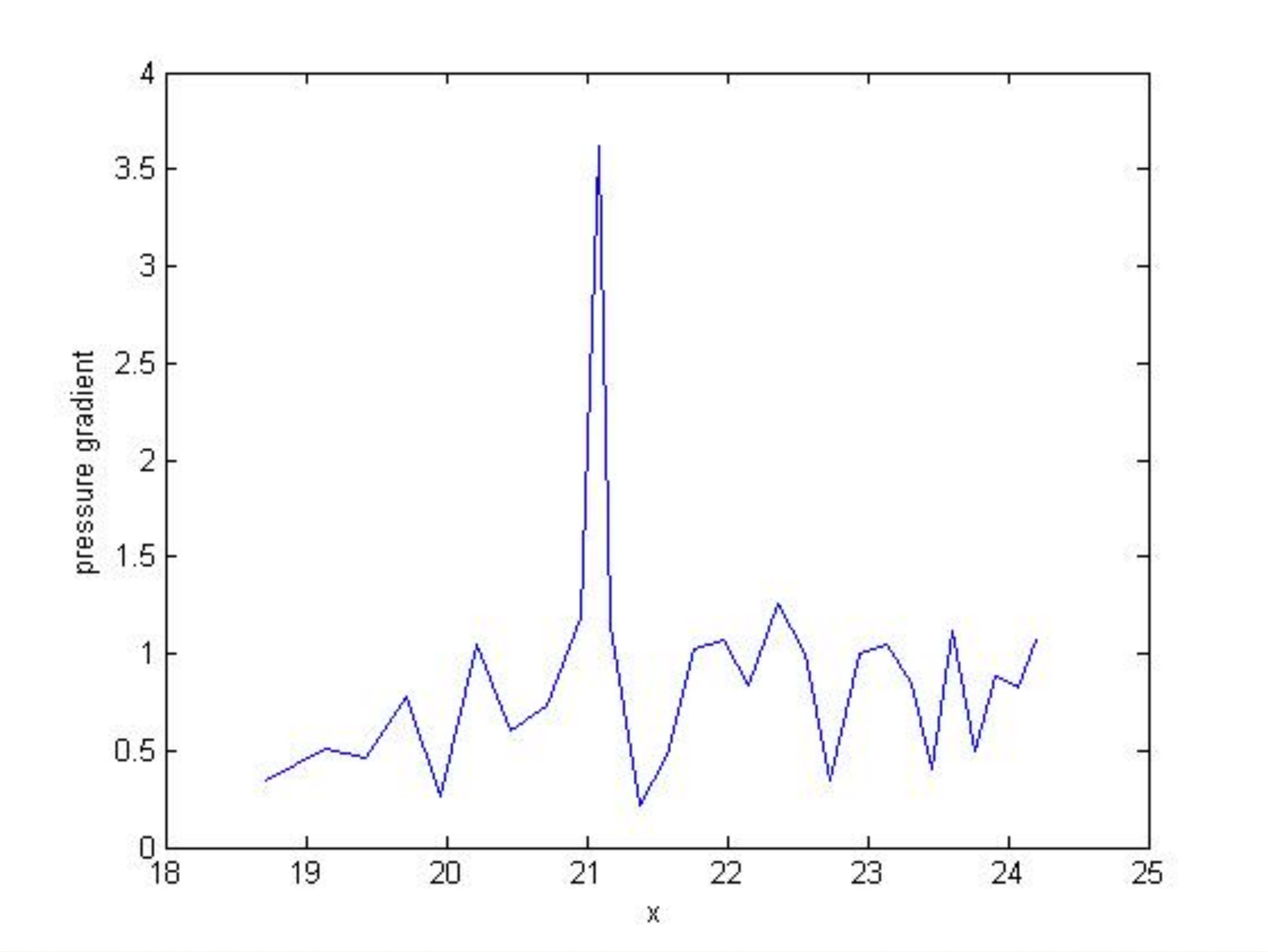
However, vorticity does not mean rotation like Blasius solution where large vorticity exists near the wall but with no rotation
Vorticity should further be further decomposed

$$\nabla\vec{V} = \frac{1}{2}\left(\nabla\vec{V} + \nabla\vec{V}^T\right) + \frac{1}{2}\left(\nabla\vec{V} - \nabla\vec{V}^T\right)$$

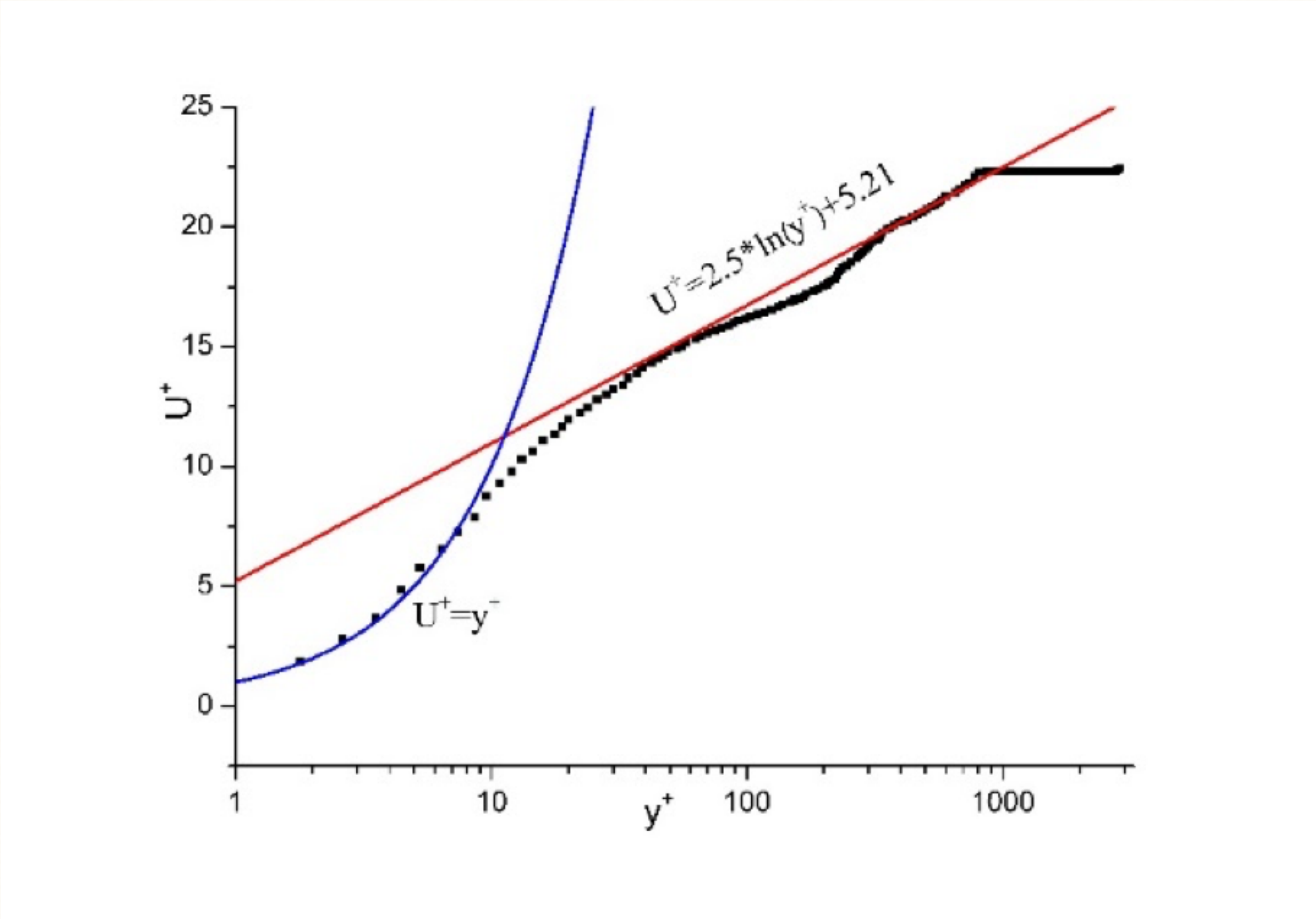
$$= \vec{\vec{\xi}} + \frac{1}{2}\left(\nabla\vec{V} - \nabla\vec{V}^T\right) \qquad (3)$$

$$d\vec{V} = d\vec{X}\cdot\vec{\vec{\xi}} - d\vec{X}\times\vec{\omega}, \text{ where } \vec{\omega} = \frac{1}{2}\nabla\times\vec{V} \qquad (4)$$

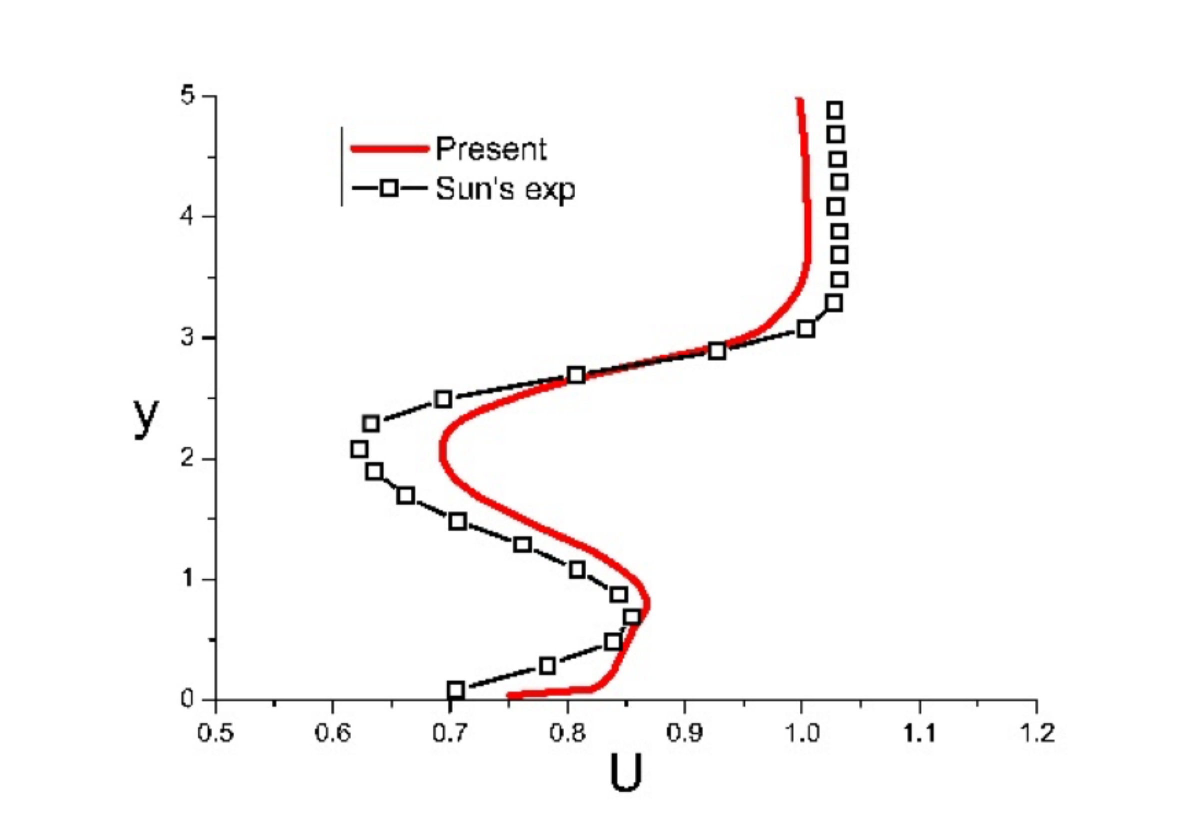
Shock



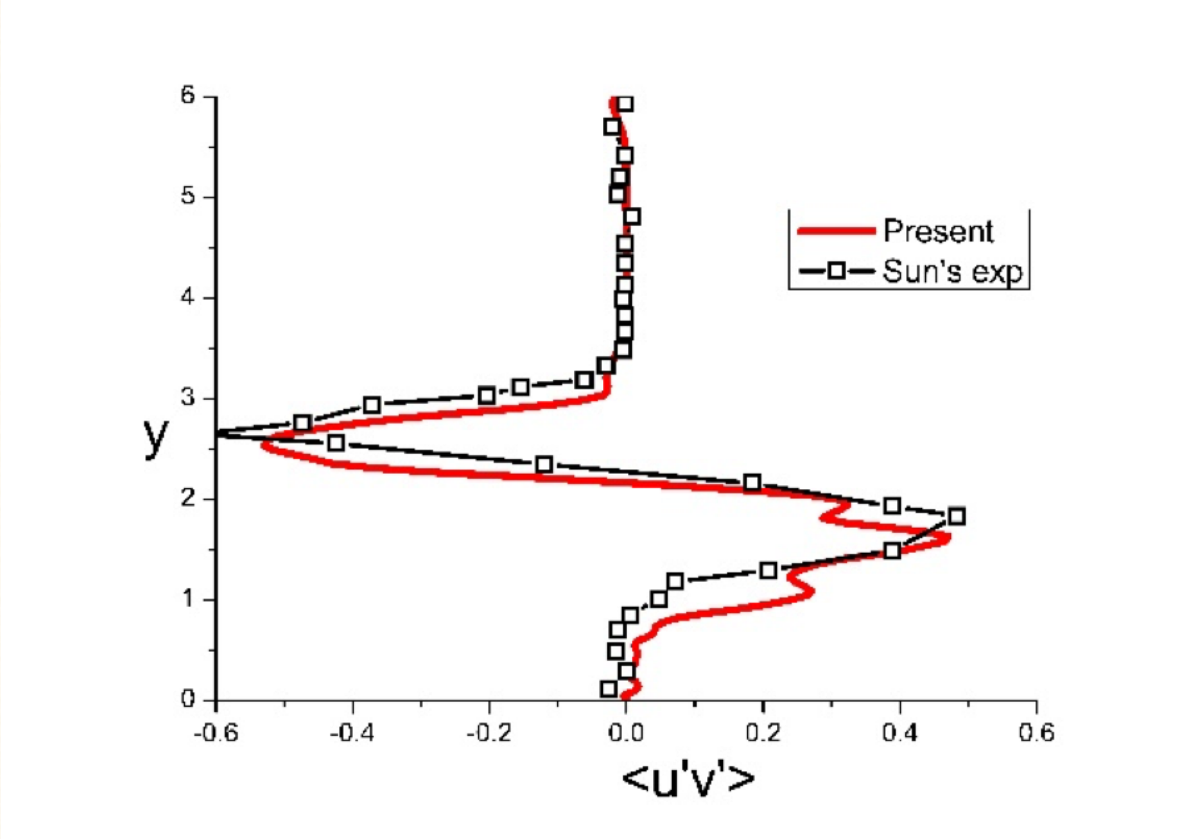
Incoming BL Profile



Mean Profile in MVG wake



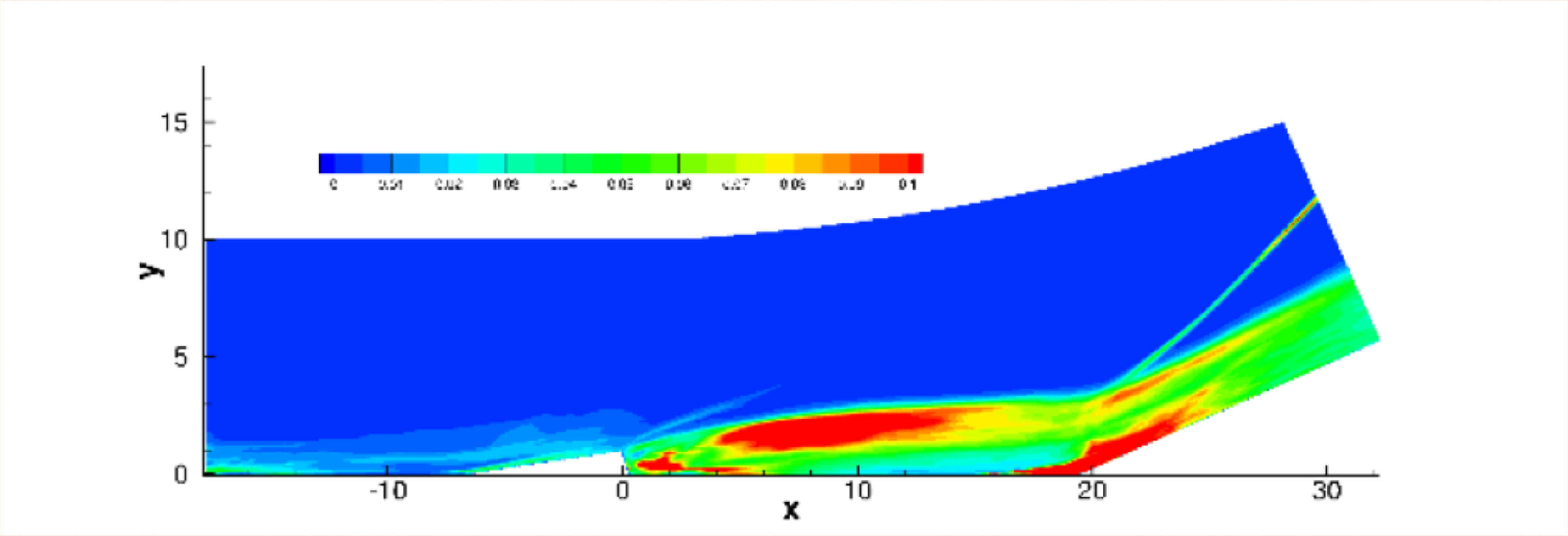
Mean Flow

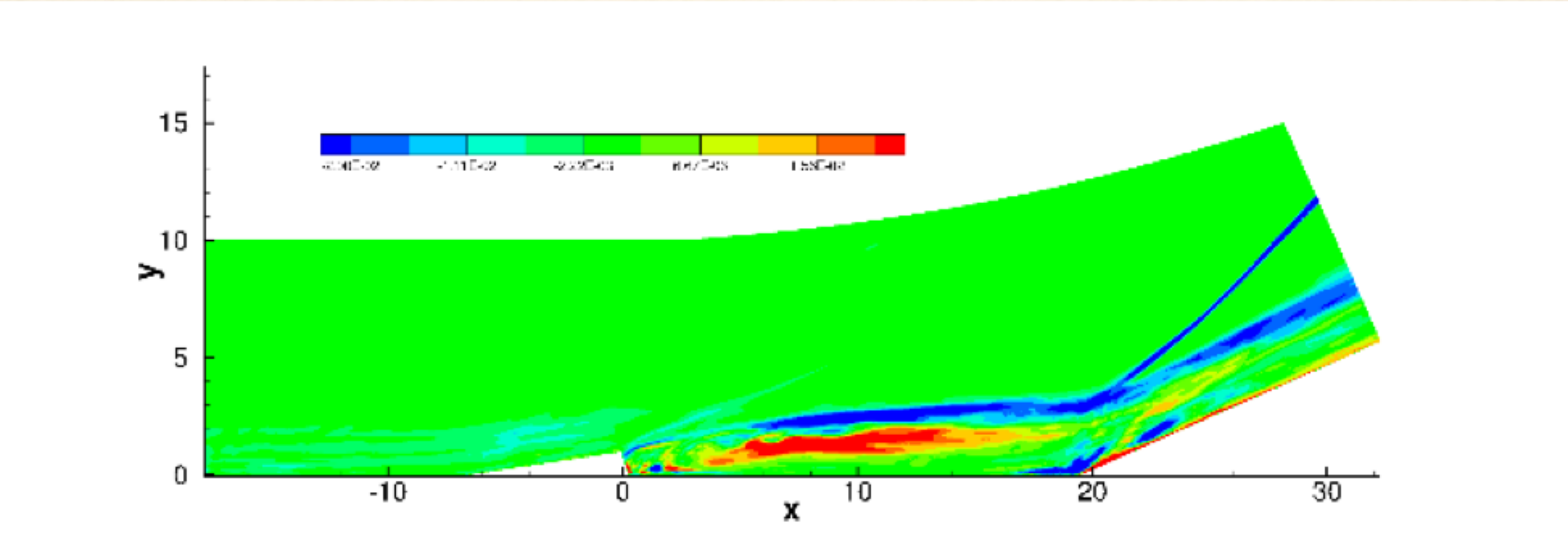


Reynolds Stress

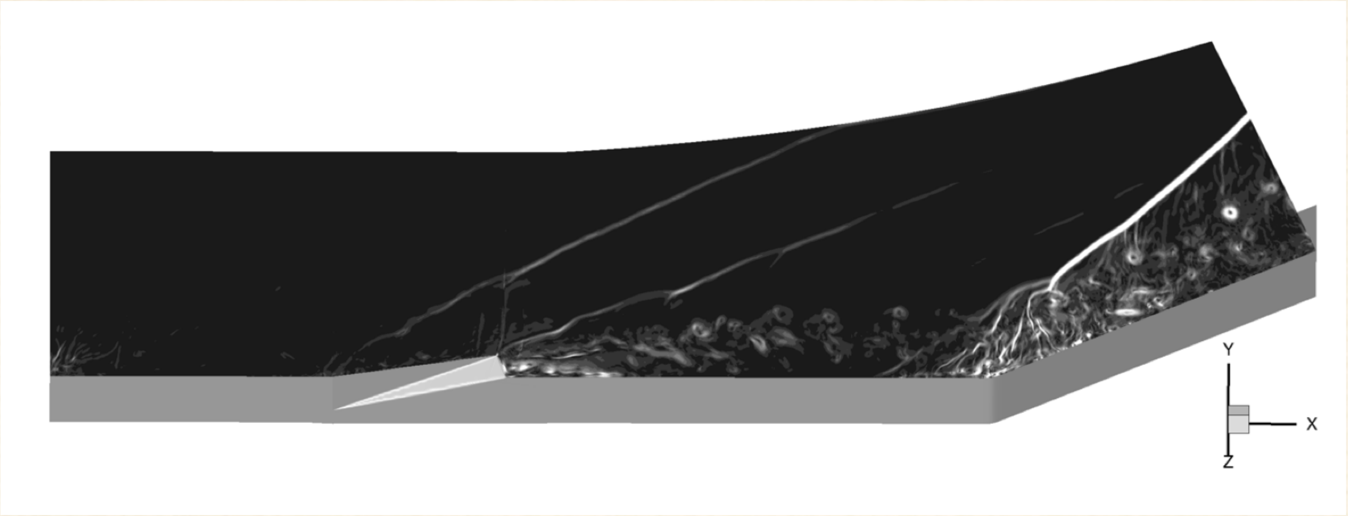
x/h=12.0

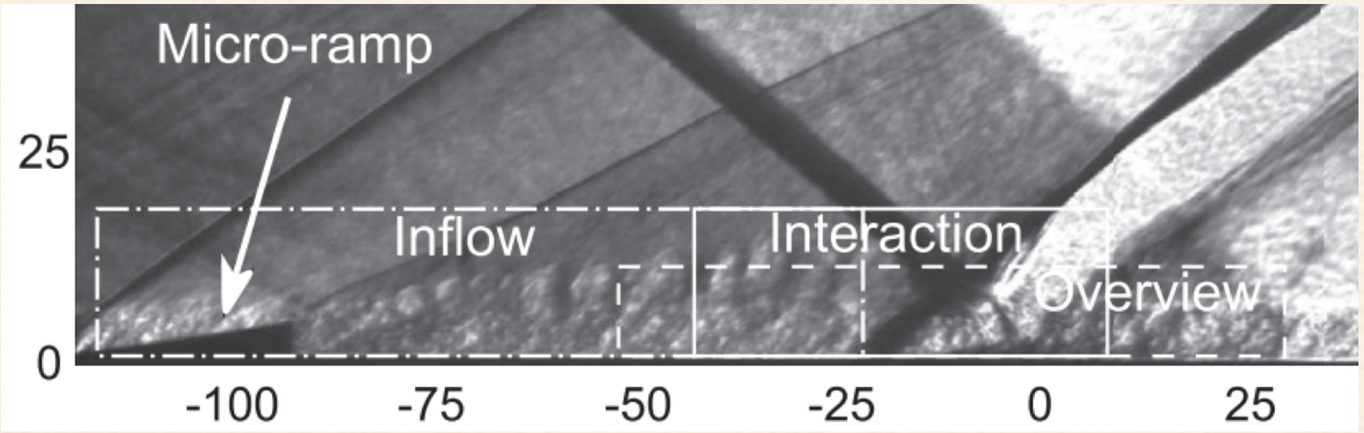
KTE & RSS

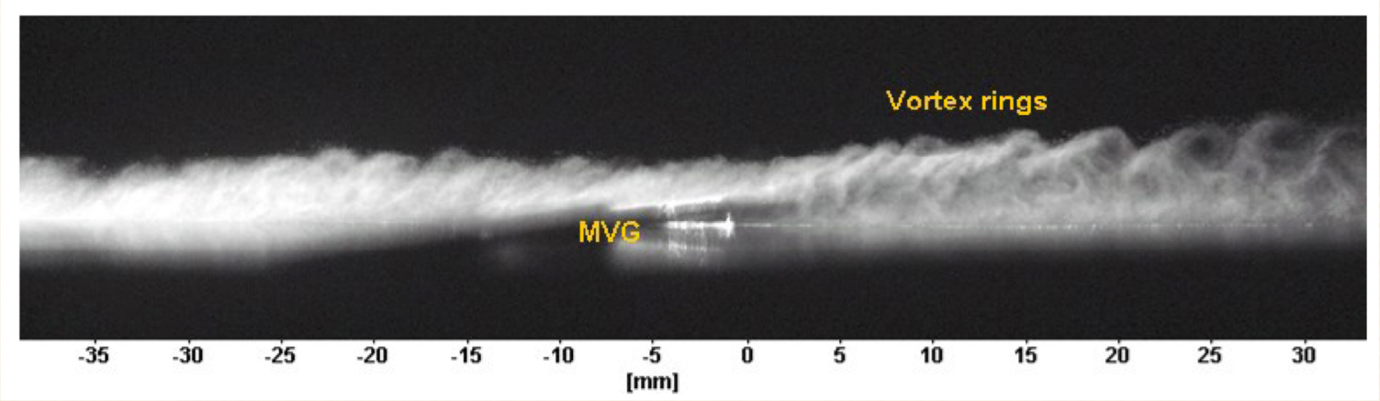


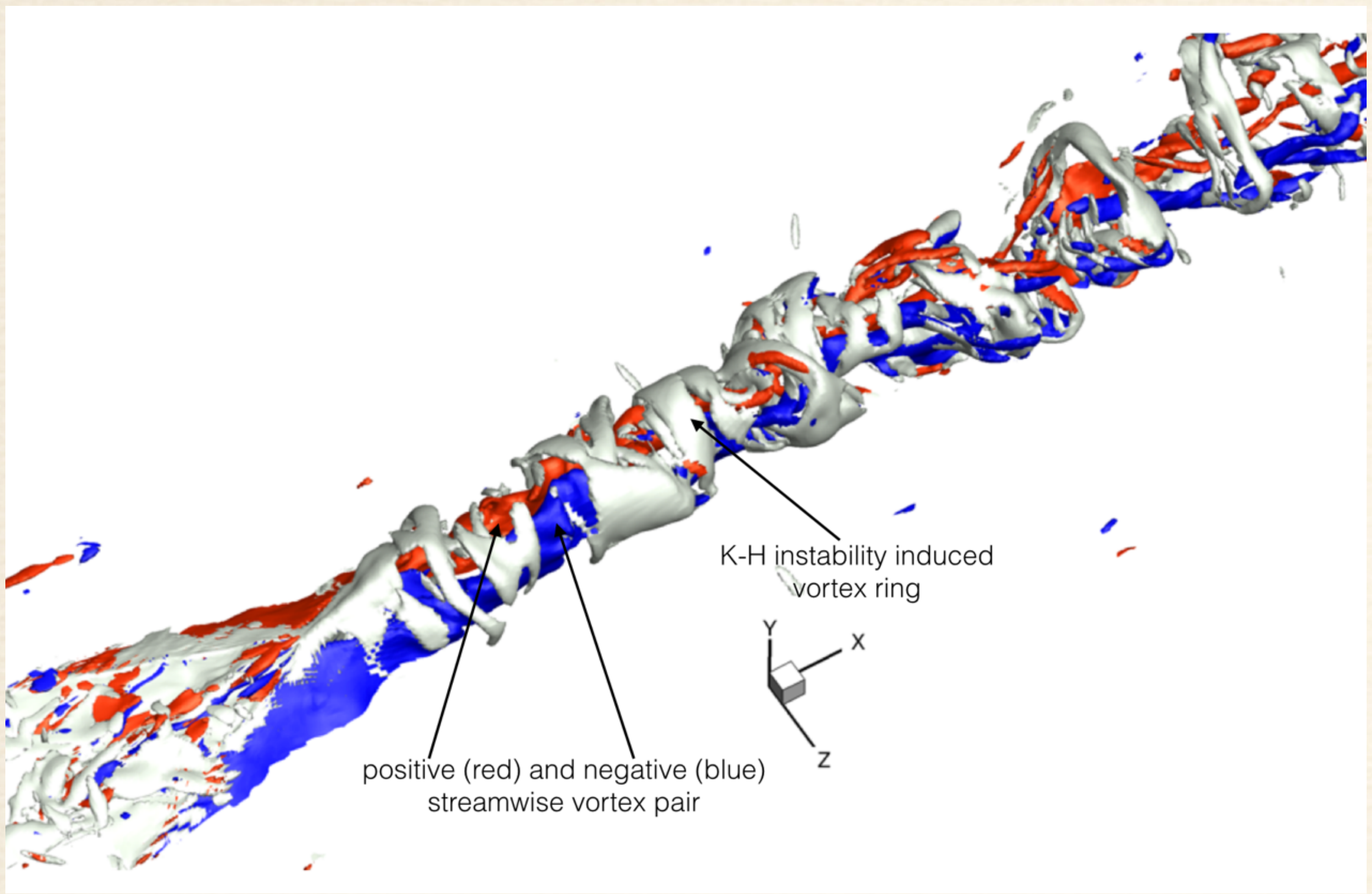


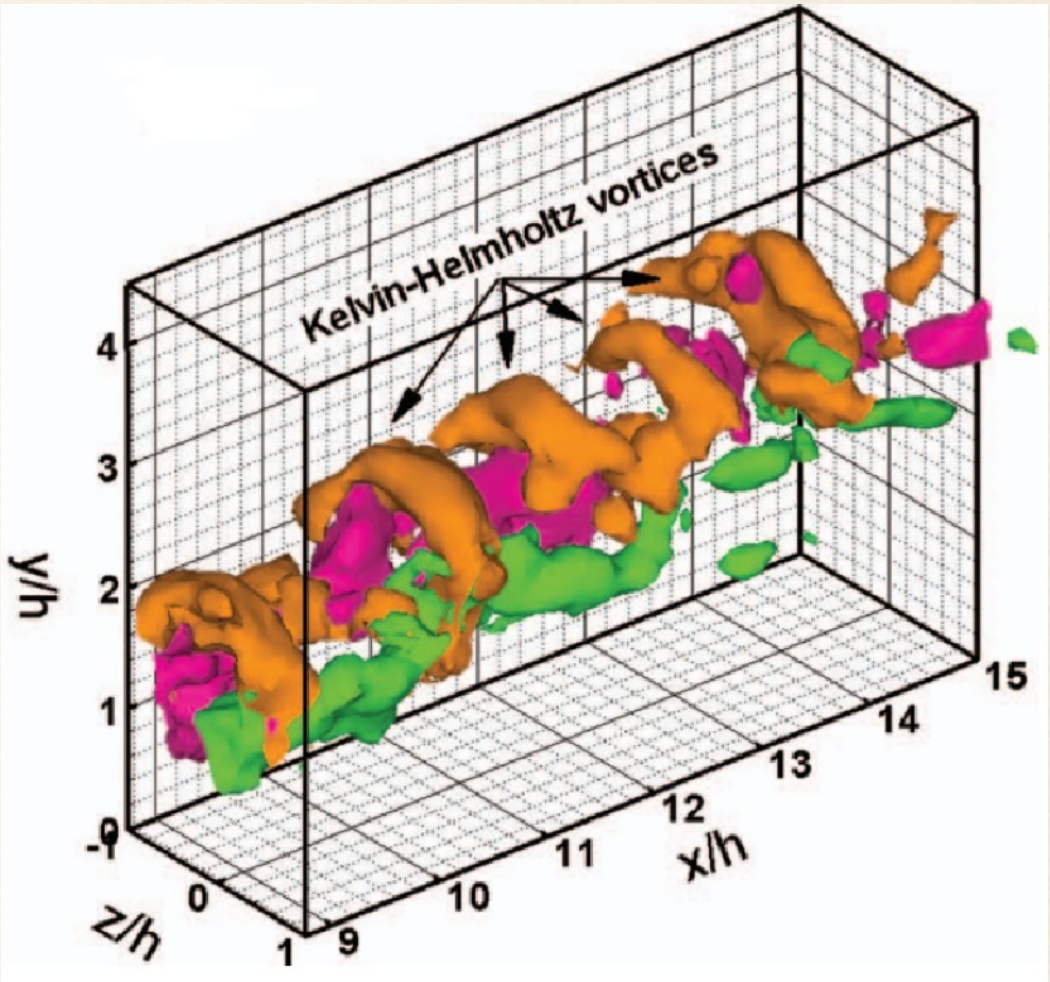
Schlieren Graph

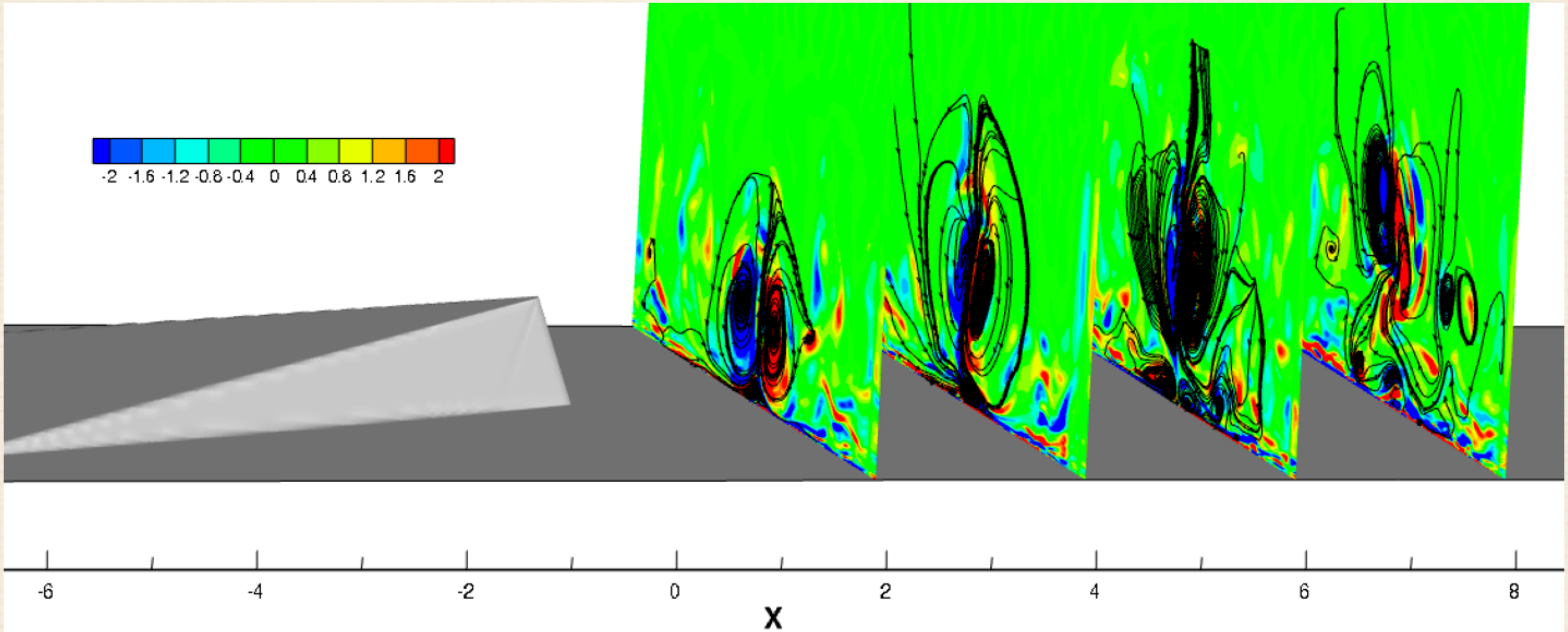


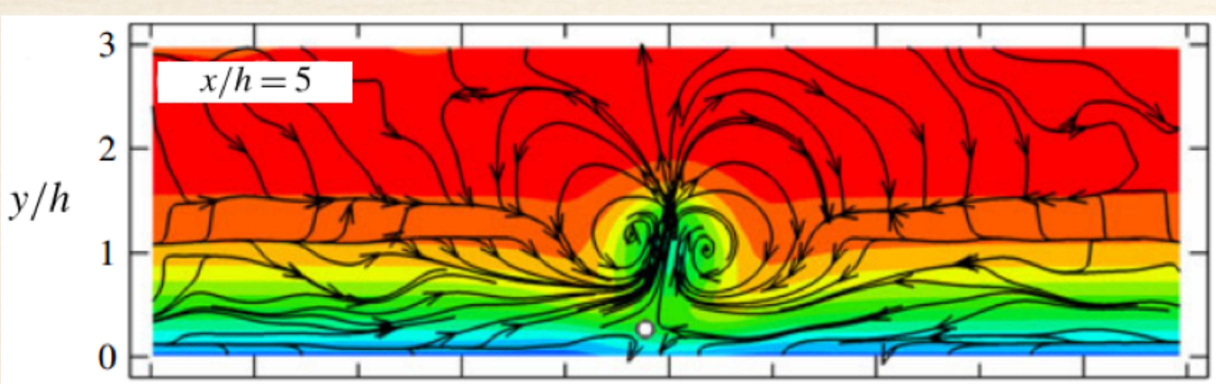


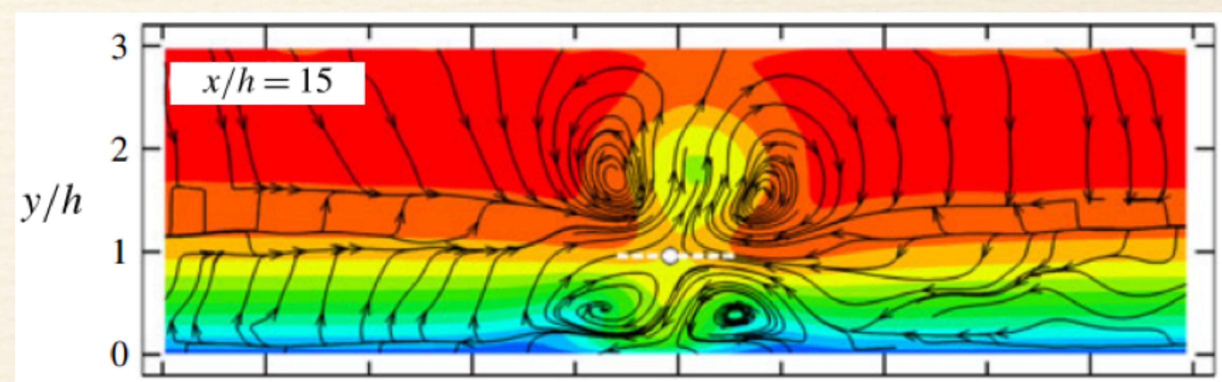



K-H instability induced
vortex ring
positive (red) and negative (blue)
streamwise vortex pair
Y
X
Z

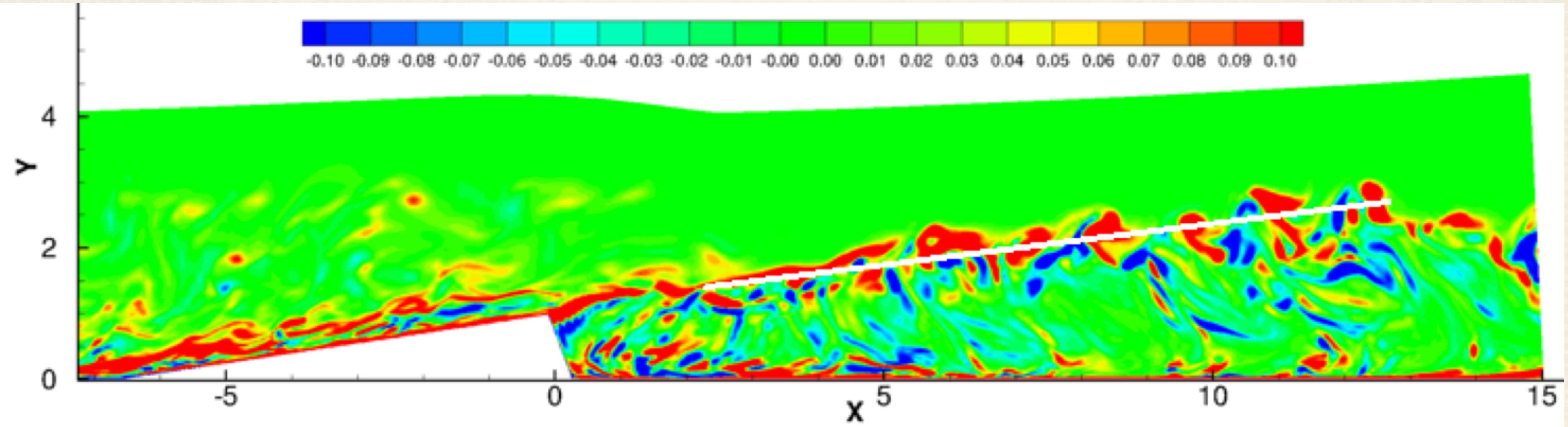

Kelvin-Helmholtz vortices
y/h
x/h
z/h
4
3
2
1
0
-1
0
1
9
10
11
12
13
14
15

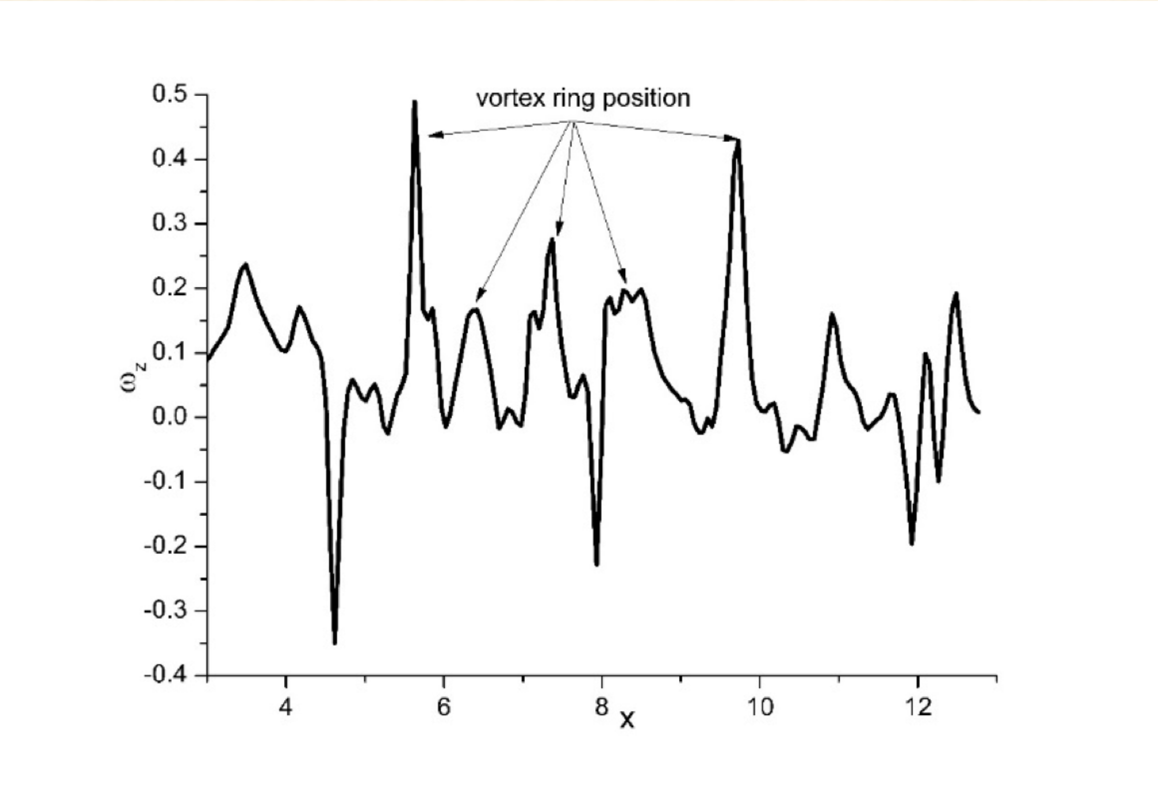

-2 -1.6 -1.2 -0.8 -0.4 0 0.4 0.8 1.2 1.6 2
-6 -4 -2 0 2 4 6 8
X


x/h = 5
y/h
0 1 2 3


x/h = 15
y/h
0 1 2 3

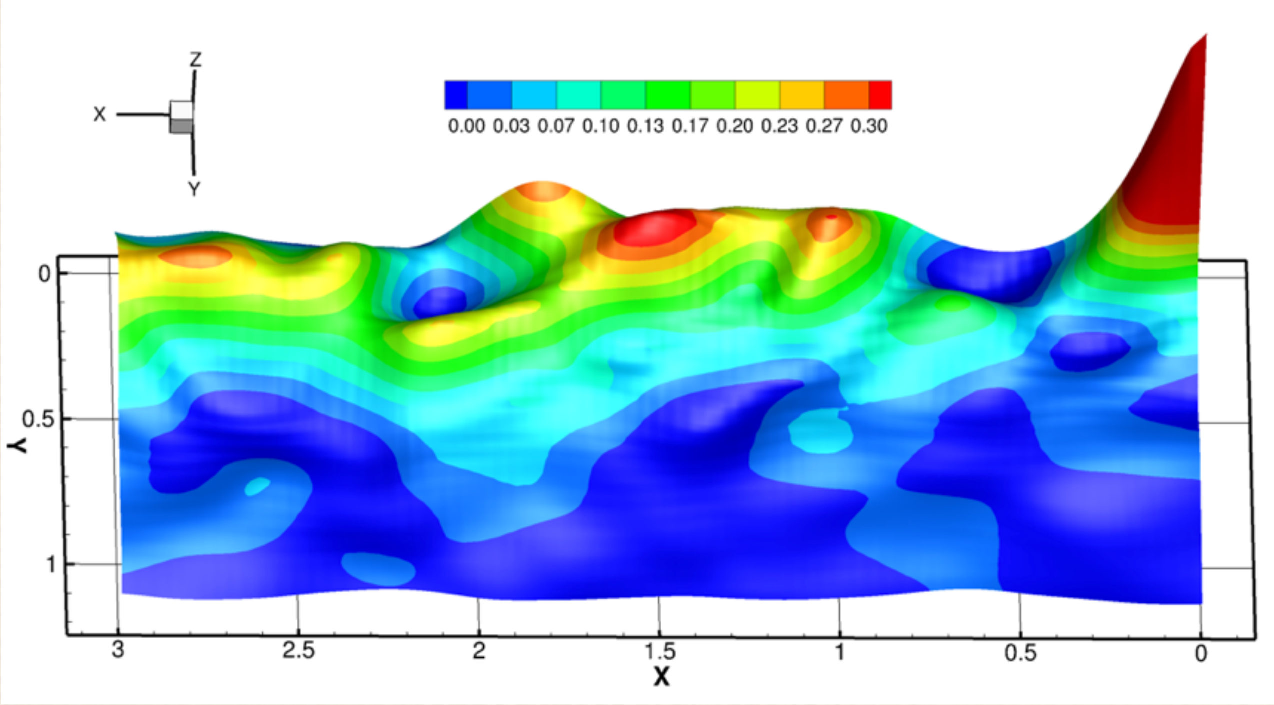
Vortex Rings Distance

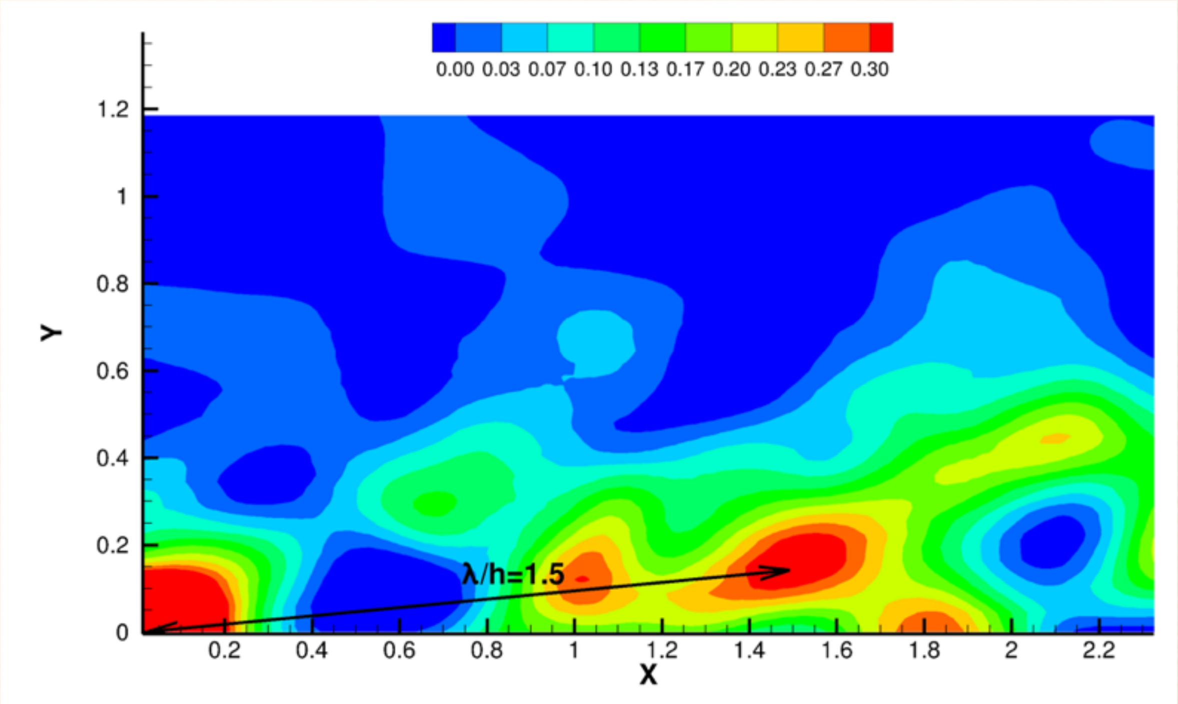




Vortex Rings Distance

$$R(\Delta x, \Delta y) = \frac{\iint \omega_z(x,y) * \omega_z(x+\Delta x, y+\Delta y)dxdy}{\iint \omega_z(x,y)^2 dxdy}$$





POD

The fluctuating velocity matrix U is calculated by substracting the mean velocities from the individual snapshots. Then the autocovariance matrix is computed as:

$$C = U^T U \tag{15}$$

The eigenvalue problem for the matrix reads as follows:

$$CA^i = \lambda^i A^i \tag{16}$$

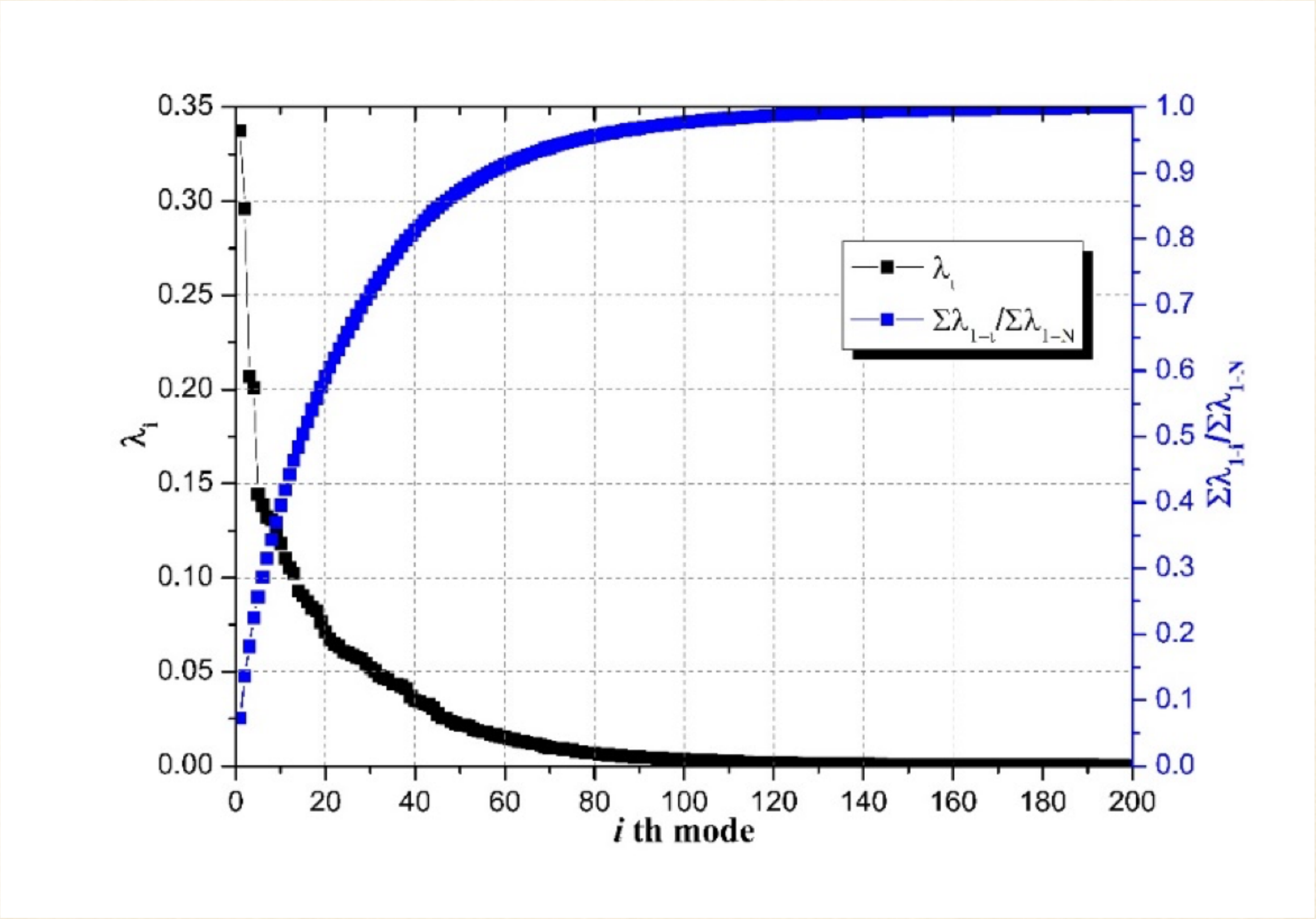
The eigenvectors are arranged according to the decreasing order of eigenvalues reflecting the energies in different POD modes.

$$\lambda^1 > \lambda^2 > \lambda^3 > \lambda^4 > \lambda^N = 0. \tag{17}$$

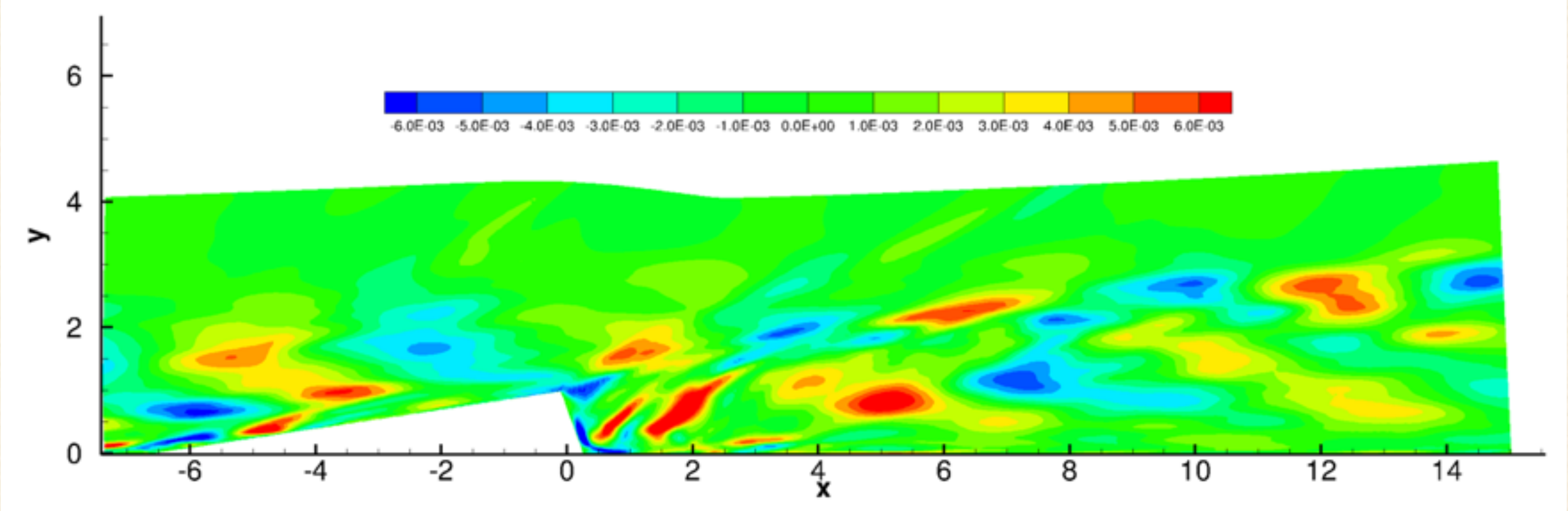
Using the ordered eigenvectors the POD modes are constructed.

$$\phi^i = \frac{\sum_{n=1}^{N} A_n^i U^n}{\| \sum_{n=1}^{N} A_n^i U^n \|}, \qquad i = 1, 2, \cdots, N. \tag{18}$$

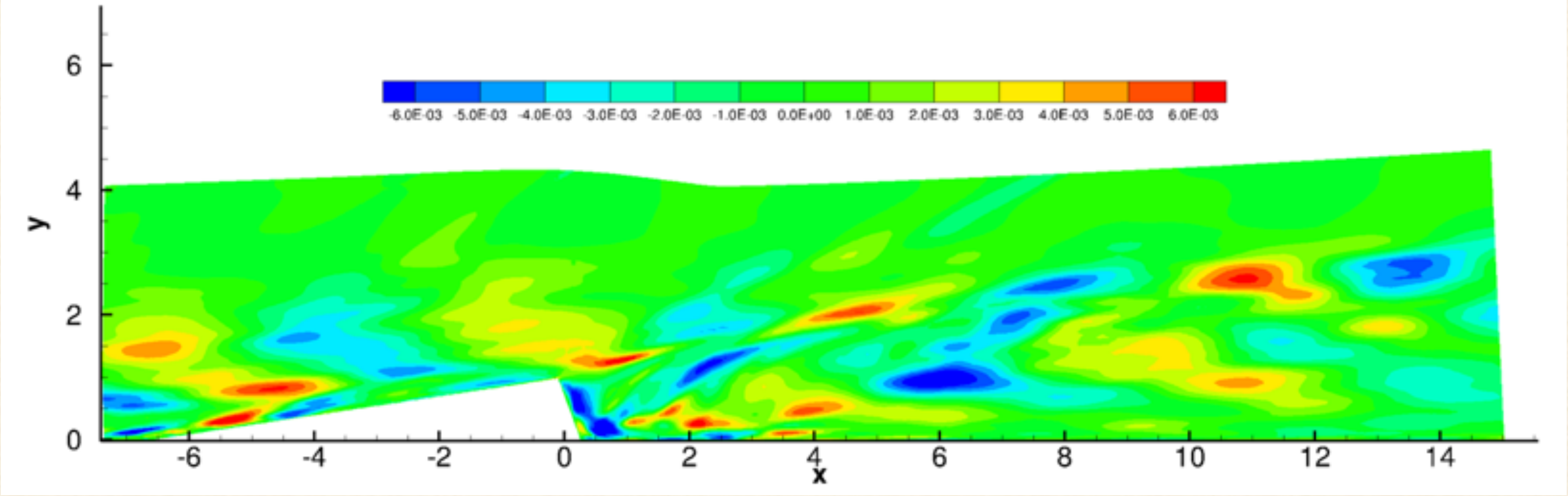
POD Eigenvalues



POD Modes

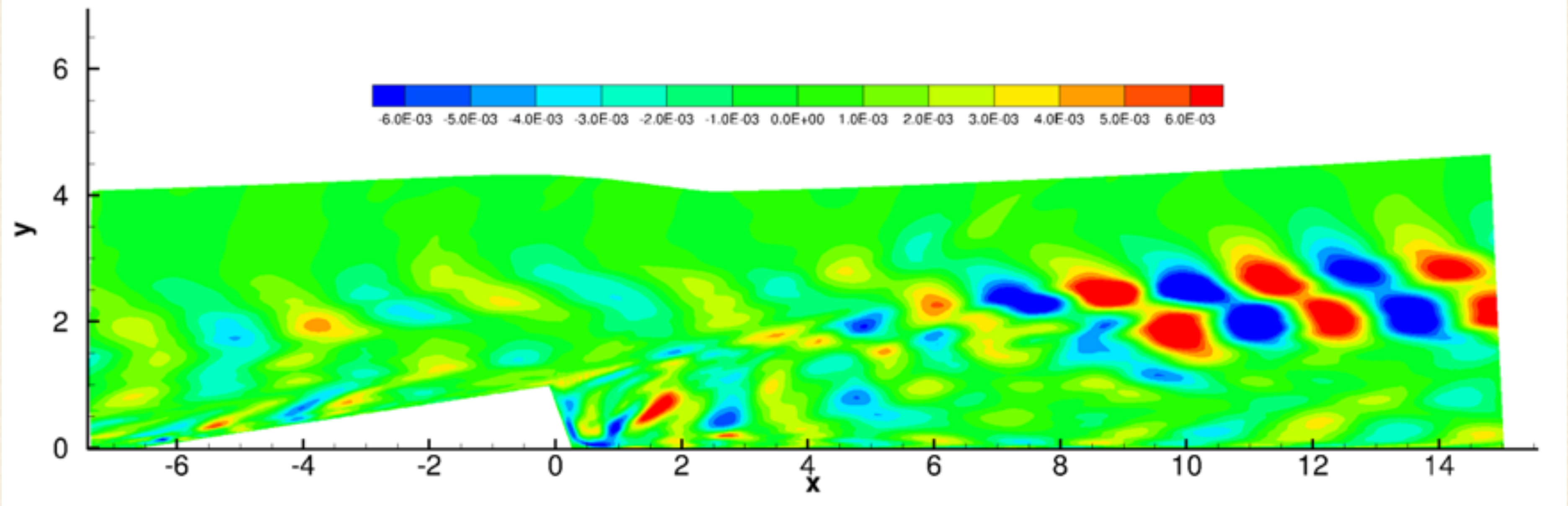


-6.0E-03 -5.0E-03 -4.0E-03 -3.0E-03 -2.0E-03 -1.0E-03 0.0E+00 1.0E-03 2.0E-03 3.0E-03 4.0E-03 5.0E-03 6.0E-03
y
6
4
2
0
-6 -4 -2 0 2 4 6 8 10 12 14
x

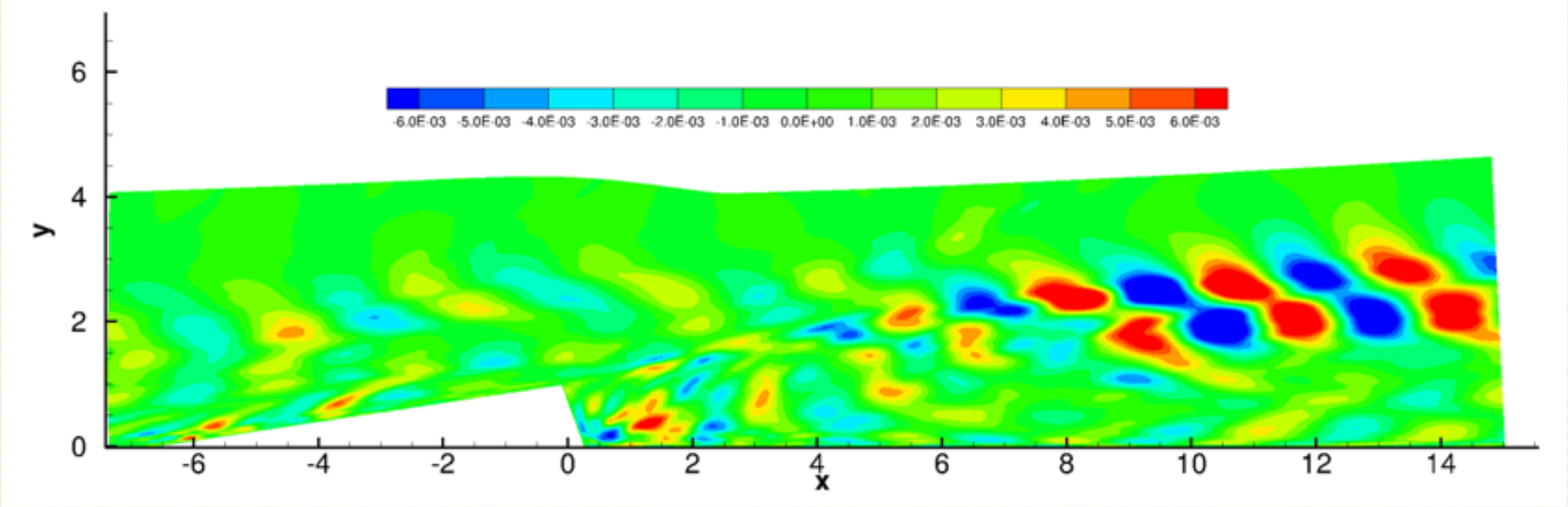


-6.0E-03 -5.0E-03 -4.0E-03 -3.0E-03 -2.0E-03 -1.0E-03 0.0E+00 1.0E-03 2.0E-03 3.0E-03 4.0E-03 5.0E-03 6.0E-03
y
6
4
2
0
-6 -4 -2 0 2 4 6 8 10 12 14
x

POD Modes

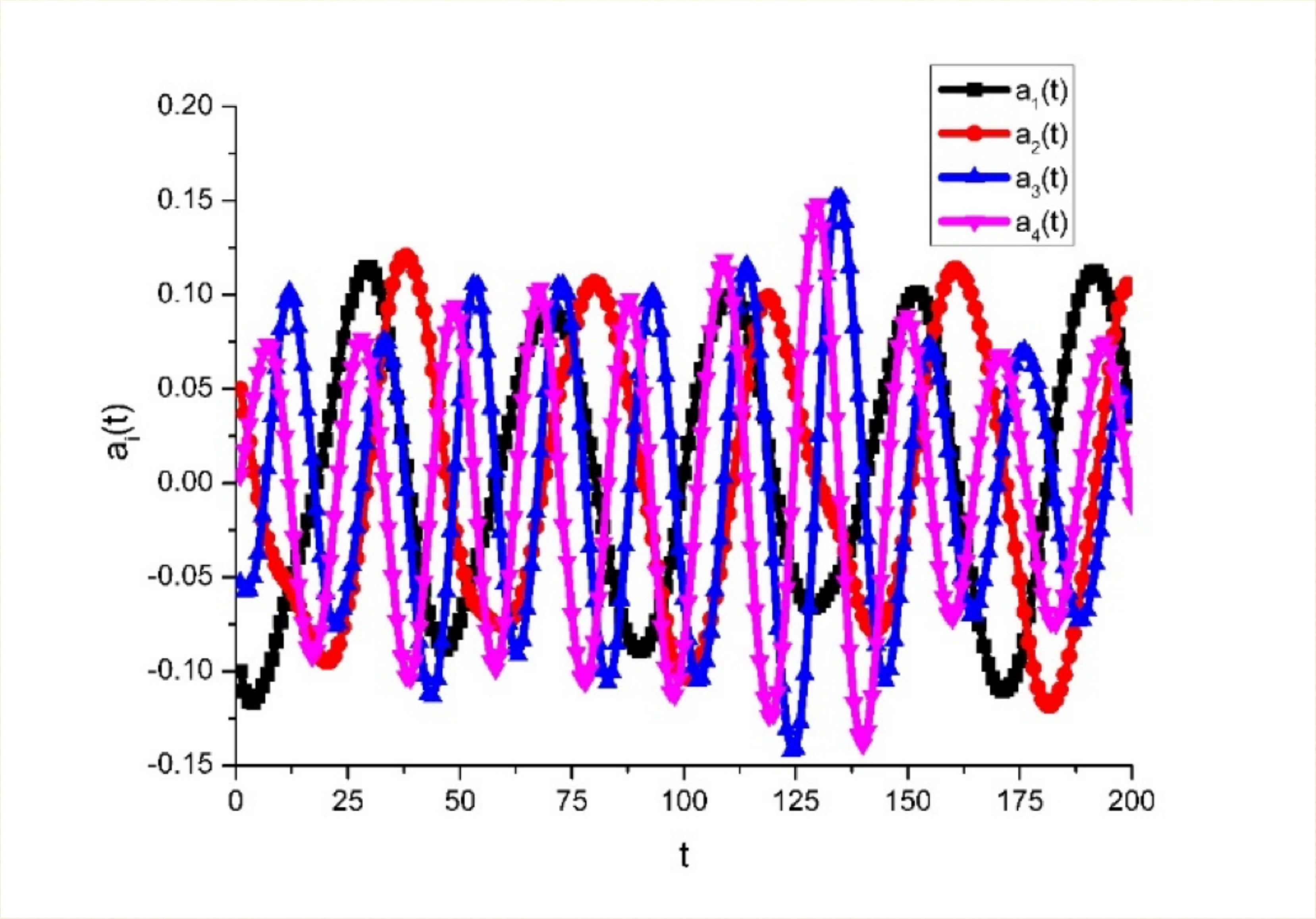


-6.0E-03 -5.0E-03 -4.0E-03 -3.0E-03 -2.0E-03 -1.0E-03 0.0E+00 1.0E-03 2.0E-03 3.0E-03 4.0E-03 5.0E-03 6.0E-03
y
6
4
2
0
-6
-4
-2
0
2
4
6
8
10
12
14
x



-6.0E-03 -5.0E-03 -4.0E-03 -3.0E-03 -2.0E-03 -1.0E-03 0.0E+00 1.0E-03 2.0E-03 3.0E-03 4.0E-03 5.0E-03 6.0E-03
y
6
4
2
0
-6
-4
-2
0
2
4
6
8
10
12
14
x

POD Time Coefficients



DMD

Calculation of fluctuating velocity matrix is not needed. But the snapshot matrix is divided into two parts.

$$V_1^{n-1} = [v_1 v_2 v_3 \quad \dots \quad v_{n-1}] \tag{19}$$

$$V^n{}_2 = [v_2 v_3 v_4 \dots v_n] \tag{20}$$

QR decomposition in economy mode is performed as :

$$[Q, R] = qr(V_1^{n-1}, 0): \tag{21}$$

Companion matrix S is calculated as :

$$S = R^{-1} Q^H V^n{}_2 . \tag{22}$$

The eigen value analysis is computed on S matrix

$$[X, D] = eig(S) \tag{23}$$

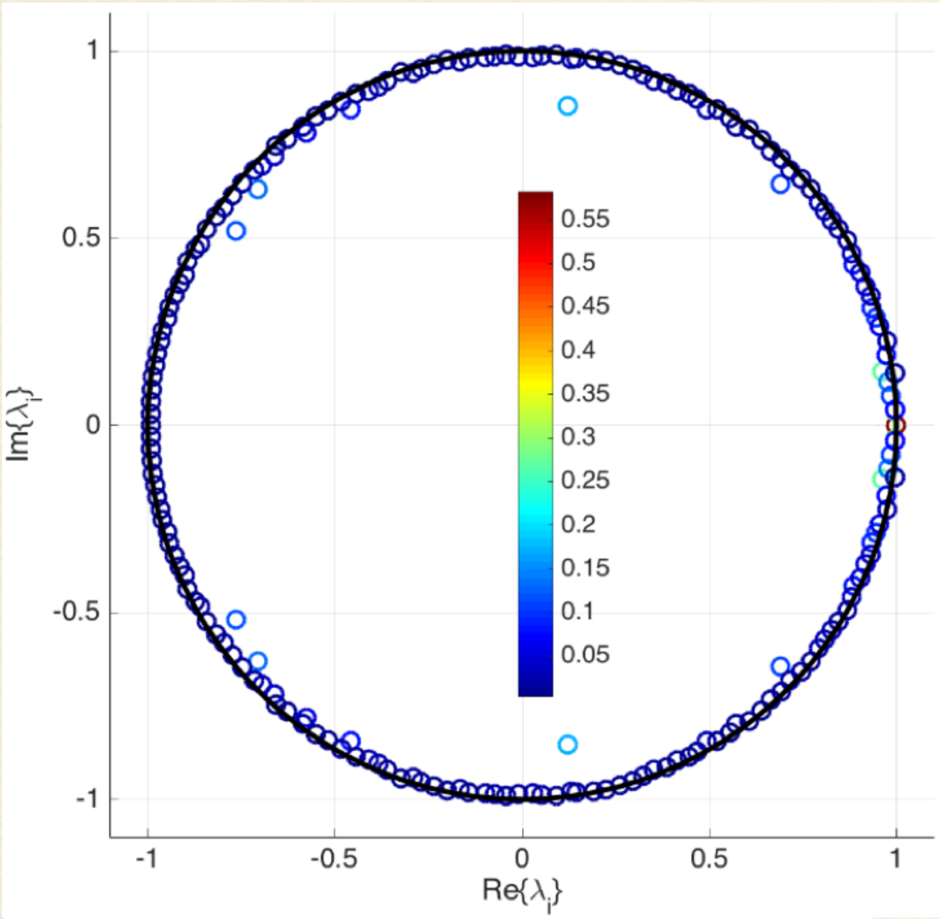
The dynamic mode spectrum is computed as :

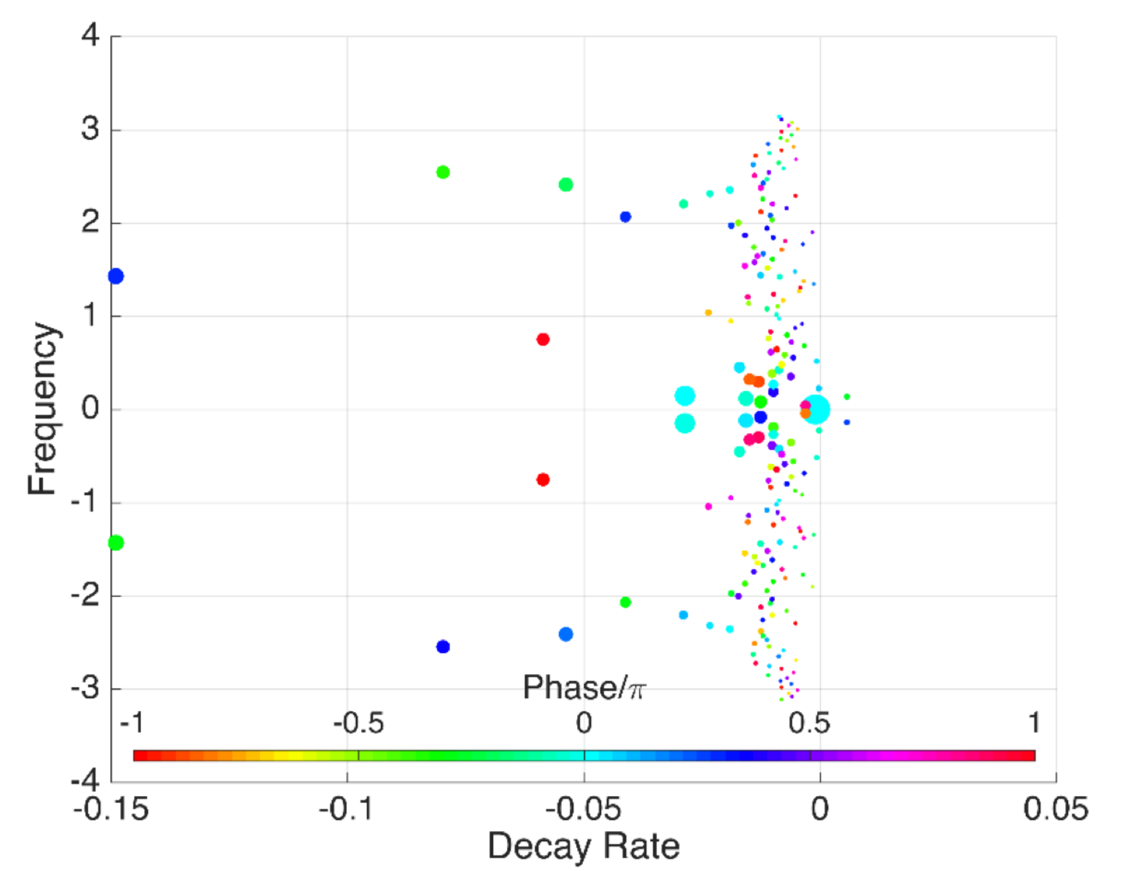
$$\lambda_j = log(D_{jj})/\delta t. \tag{24}$$

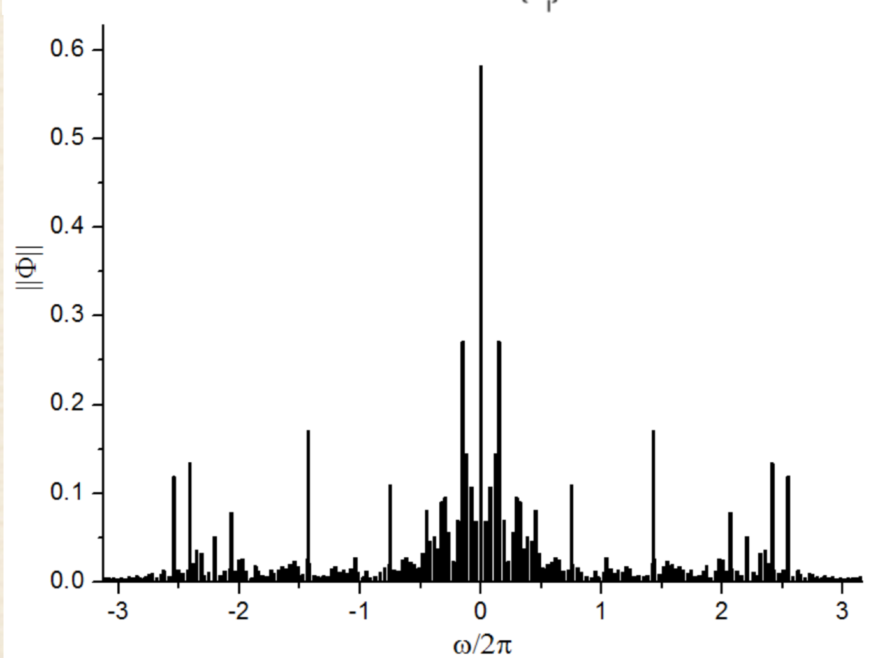
δt, is the time interval between the snapshots. The Dynamic modes can be computed as follows:

$$DM_j = V^{N-1}\, X(:, j) \tag{25}$$

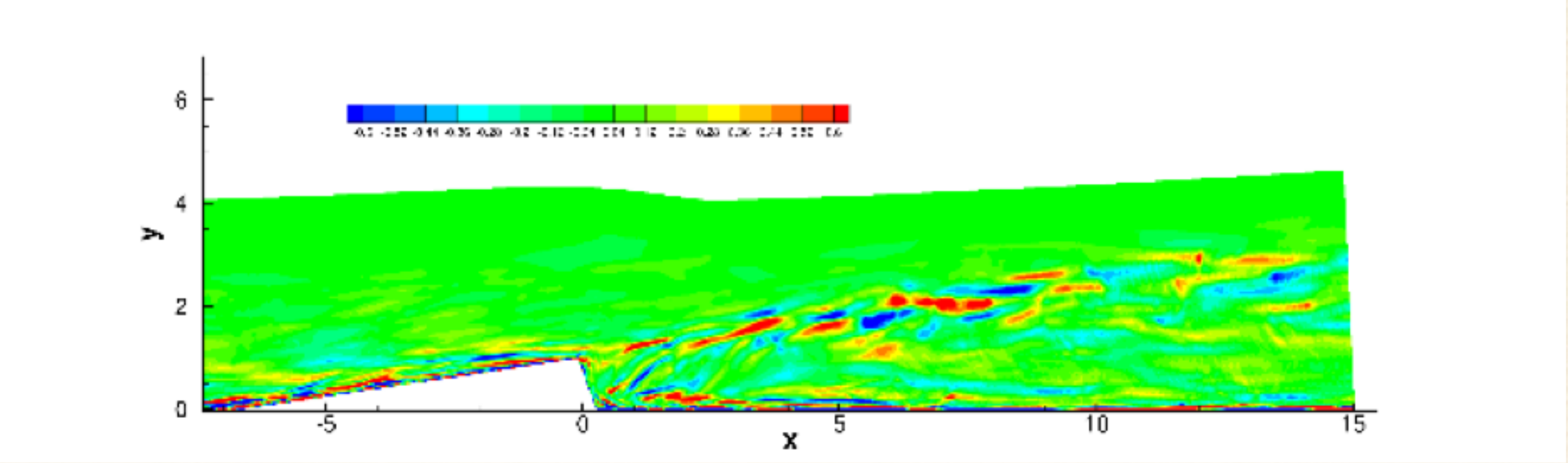
DMD Spectra


Re{λ_i}
Im{λ_i}
0.55
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05

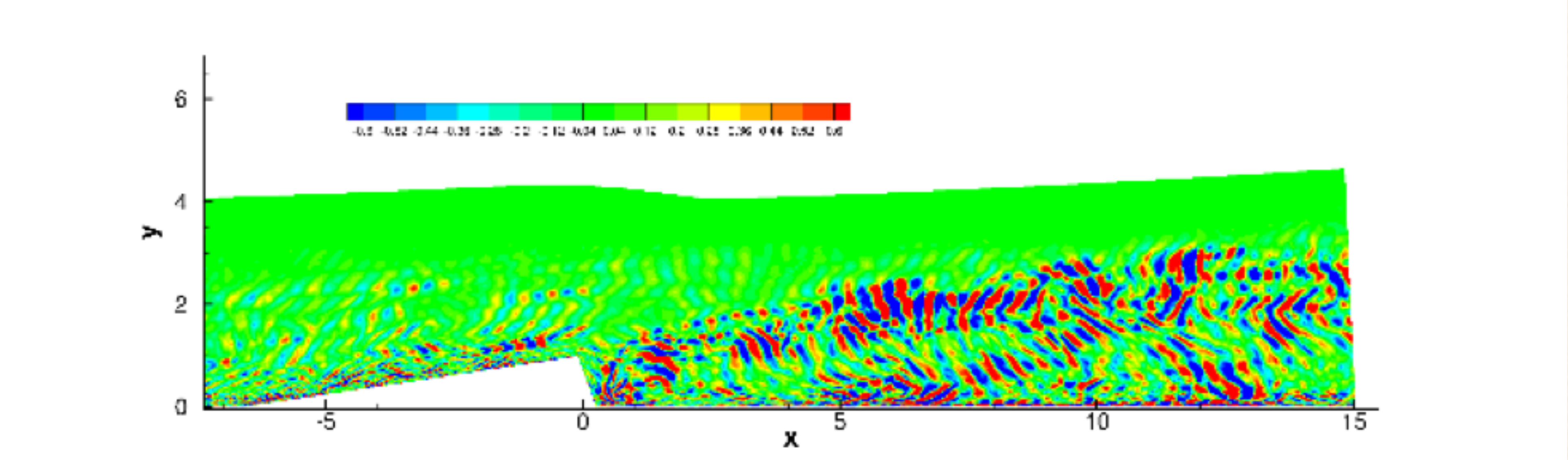

Frequency
Phase/π
Decay Rate


||Φ||
ω/2π

DMD Modes

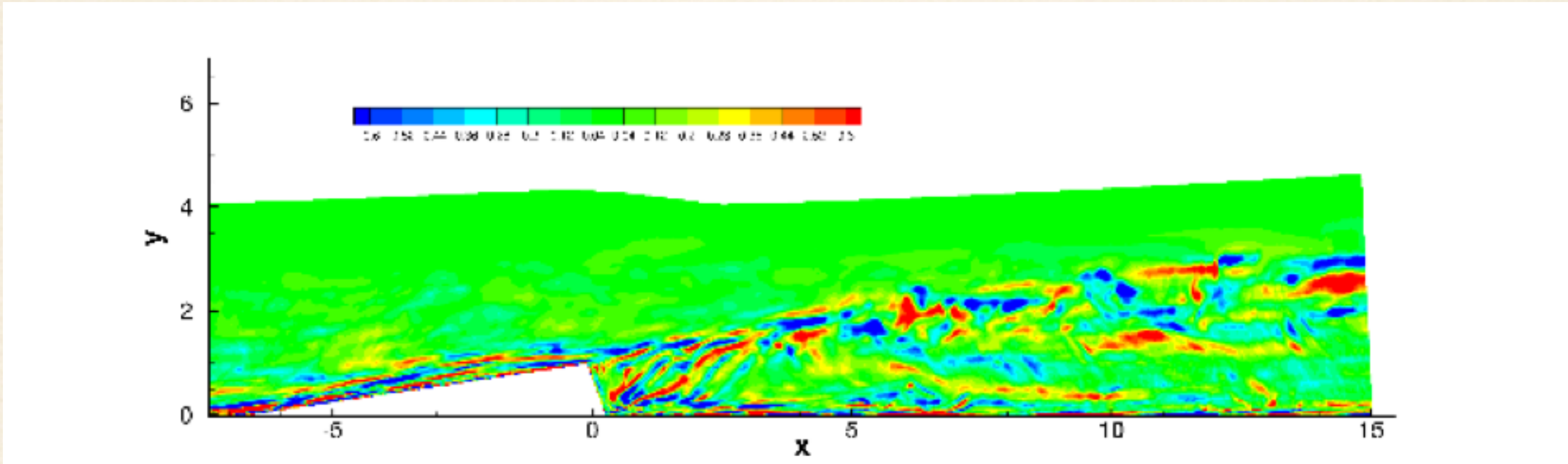


$\mu_r = -0.0286, \mu_i = 0.1488$

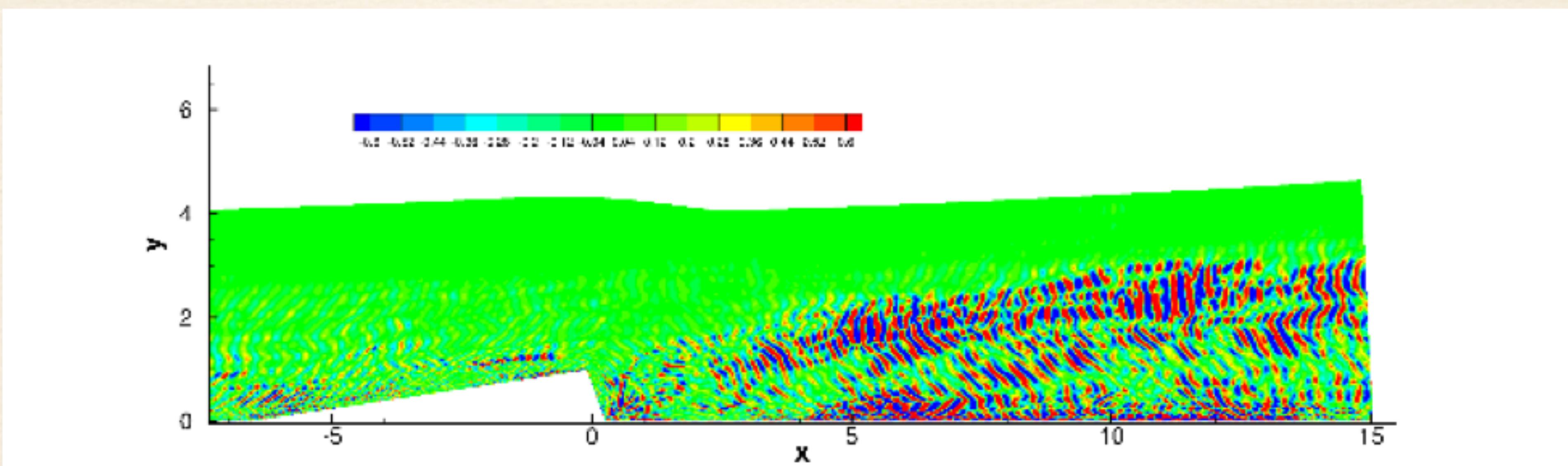


$\mu_r = -0.1490, \mu_i = 1.4301$

DMD Modes



$$\mu_r = -0.0157, \mu_i = 0.1185$$



$$\mu_r = -0.0538, \mu_i = 2.4126$$